

# Applied Mechanics Reviews

*A Critical Review of the World Literature in Applied Mechanics*

A. W. WUNDHEILER, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

## GENERAL

Theoretical and Experimental Methods . . . 241  
Mechanics (Dynamics, Statics, Kinematics) . . . 242

## MECHANICS OF SOLIDS

Gyroscopics, Governors, Servos . . . 242  
Vibrations, Balancing . . . 243  
Wave Motion, Impact . . . 244  
Elasticity Theory . . . 244  
Experimental Stress Analysis . . . 245  
Rods, Beams, Shafts, Springs, Cables,  
etc. . . . . 245  
Plates, Disks, Shells, Membranes . . . 245  
Buckling Problems . . . 246  
Structures . . . 246  
Rheology (Plastic, Viscoplastic Flow) . . . 247  
Failure, Mechanics of Solid State . . . 248  
Design Factors, Meaning of Material Tests . . . 249  
Material Test Techniques . . . 249  
Mechanical Properties of Specific Mate-  
rials . . . 250  
Mechanics of Forming and Cutting . . . 251

## MECHANICS OF FLUIDS

Hydraulics; Cavitation; Transport . . . 252  
Incompressible Flow: Laminar; Viscous. . . 253  
Compressible Flow, Gas Dynamics . . . 253  
Turbulence, Boundary Layer, etc. . . . 256  
Aerodynamics of Flight; Wind Forces . . . 257  
Aeroelasticity (Flutter, Divergence, etc.). . . 258  
Propellers, Fans, Turbines, Pumps, etc. . . 258  
Flow and Flight Test Techniques . . . 259

## HEAT

Thermodynamics . . . . . 259  
Heat Transfer; Diffusion . . . . . 260

## MISCELLANEOUS

Acoustics . . . . . 261  
Ballistics, Detonics (Explosions) . . . 261  
Soil Mechanics, Seepage. . . . . 261  
Geophysics, Meteorology, Oceanography. . . 262  
Lubrication; Bearings; Wear . . . . . 264  
Marine Engineering Problems . . . . . 264

Published by The American Society of Mechanical Engineers

November 1949

Vol. 2, No. 11

# Applied Mechanics Reviews

Published Monthly by The American Society of Mechanical Engineers  
at Easton, Pa., with the co-operation of

THE OFFICE OF NAVAL RESEARCH

AMERICAN SOCIETY OF CIVIL ENGINEERS  
INSTITUTE OF THE AERONAUTICAL SCIENCES  
SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS

ILLINOIS INSTITUTE OF TECHNOLOGY

THE ENGINEERING INSTITUTE OF CANADA

THE ENGINEERING FOUNDATION

AMERICAN INSTITUTE OF PHYSICS  
AMERICAN MATHEMATICAL SOCIETY  
THE INSTITUTION OF MECHANICAL ENGINEERS

## OFFICERS OF ASME:

J. M. TODD, *President*  
K. W. JAPPE, *Treasurer*      C. E. DAVIES, *Secretary*

## ASME MANAGING COMMITTEE:

G. B. PEGRAM, *Chairman*  
H. L. DRYDEN      J. S. THOMPSON  
M. GOLAND, *ASME Applied Mechanics Division*

## ADVISORY BOARD:

R. E. PETERSON (ASME), *Chairman*      R. D. MINDLIN (SESA), *Secretary*  
R. COURANT (AMS)      G. R. RICH (ASCE)      F. V. HUNT (AIP)      H. W. SWIFT (IME)  
L. H. DONNELL (IIT)      K. S. M. DAVIDSON (IAS)      V. MORKOVIN (ONR)      J. J. GREEN (EIC)

## EDITORIAL STAFF:

A. W. WUNDHEILER, *Editor*      T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*  
A. J. DURELLI,      J. E. GOLDBERG,      E. F. LYPE,      D. R. MAZKEVICH,      E. VEY,  
ISAO IMAI (Japan),      K. ZARANKIEWICZ (Poland), *Assistant Editors*

*Editorial Office:* APPLIED MECHANICS REVIEWS, Illinois Institute of Technology, Chicago 16, Ill., USA.

*Subscription and Production Office:* The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., USA.

**PHOTOSTAT SERVICE:** Photostatic copies of all articles reviewed in this issue will be provided by the editors on request. Orders should specify the number and issue of APPLIED MECHANICS REVIEWS review; should be addressed to APPLIED MECHANICS REVIEWS, Illinois Institute of Technology, Chicago 16, Ill., USA; and should be accompanied by a remittance to cover cost, amounting to 25 cents for each page of the article photostated. Minimum charge \$1.00. (Applicant assumes responsibility for questions of copyright arising from this copying and the use made of copies. Copyright material will not be reproduced beyond recognized "fair use" without consent of copyright owner.)

**ABBREVIATIONS:** In abbreviating the titles of periodicals the standard employed in the World List of Scientific Periodicals, Oxford University Press, London, has been followed. In this usage prepositions, articles, and other unimportant words are omitted. Enough of each word is retained to make its meaning obvious, except in the case of common periodical designations such as: J. (Journal); G. (Giornale); C. R. (Comptes Rendus); Z. (Zeitschrift); R. C. (Rendiconti).

Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

APPLIED MECHANICS REVIEWS, November 1949, Vol. 2, No. 11. Published monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., USA. The editorial office is located at the Illinois Institute of Technology, Chicago 16, Ill., USA. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., USA. Cable address "Dynamic," New York. Price \$1.50 per copy, \$12.50 a year; to members of ASME and co-operating societies \$0.75 per copy, \$9 a year. Changes of address must be received at Society headquarters three weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. . . . Copyrighted, 1949, by The American Society of Mechanical Engineers.

# Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

November 1949

Vol. 2, No. 11

## Theoretical and Experimental Methods

(See also Revs. 1355, 1360, 1362, 1380, 1413, 1449)

1345. Hilda Geiringer, "On the solution of systems of linear equations by certain iteration methods," *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 365-393.

This is in the main an expository article. The author gives a general iteration scheme which contains most of the known procedures as special cases. She states that one of her objectives is to "state simply and clearly some facts which are occasionally mentioned in a vague or even inaccurate way" concerning the solution of linear equations by iteration, and she has succeeded admirably in attaining this objective. Considerable space is devoted to questions of convergence and to the role of the "starting point" for the various iteration schemes. A number of well-chosen examples illustrate key points which are raised in the general treatment. The procedures proposed seem ready-made for modern high-speed computing devices.

Benjamin Epstein, USA

1346. H. de Sloovere, "On the Lagrangian multipliers in the solution of problems in the calculus of variations" (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 10, pp. 735-747.

This paper shows in a very general manner that Th. de Donder's method of using Lagrangian multipliers for solving extremal problems in the calculus of variations gives precisely the same results as the classical method.

P. C. Dunne, England

1347. G. F. Carrier, "A generalization of the Wiener-Hopf technique," *Quart. appl. Math.*, Apr. 1949, vol. 7, pp. 105-109.

The integral equation of Wiener-Hopf is extended to include a more general class of kernels built up from function transforms. A particular example is discussed where the kernel arises naturally as the Green's function for a certain partial differential equation. In this case the Green's function can be represented by means of Hankel transforms, and the solution is effected by taking Hankel transforms of the integral equation and making the familiar analytical continuation argument of Wiener-Hopf to evaluate the unknown function.

Stephen H. Crandall, England

1348. P. Groen, "Exact solution of eigenvalue problems arising in connection with the study of certain hydrodynamical or quantum-mechanical wave phenomena" (in English), *Appl. sci. Res. Sec. A*, 1948, vol. A 1, no. 3, pp. 225-236.

In connection with the study of internal gravity waves the author has encountered the equation, of Schrödinger type,  $\phi'' + \{f(x) - m^2\} \phi = 0$ . He shows it is possible to solve this equation analytically when  $f(t)$  is of the type  $a[\cosh(x/c)]^{-2} + b[\sinh(x/c)]^{-2}$ . He discusses the eigenvalue problems connected with the boundary condition  $\phi(x) \rightarrow 0$  for  $|x| \rightarrow \infty$ . By a simple substitution the equation becomes a hypergeometric equation. The behavior of solutions for  $|x|$  large and  $|x|$  small is investigated, the latter being needed for eventual smooth joining of two

solutions at  $x = 0$ . Two general and two specific cases are considered in an exhaustive and elegant discussion.

P. Le Corbeiller, USA

1349. André Kovacs, "On the experimental determination of the specific energy loss by the method of coupled pendulums" (in French), *C. R. Acad. Sci. Paris*, Nov. 15, 1948, vol. 227, pp. 1019-1020.

This paper describes the technique of determining damping constants for a material by the use of two pendulums coupled elastically with the test specimen. Observations are made of the maximum and minimum amplitudes of the displacements of the pendulums for any phase angle between the positions of the pendulums. The amplitudes must be observed twice, with the number of oscillations between the amplitudes being recorded.

N. M. Newmark, USA

1350. G. W. Housner and G. D. McCann, "The analysis of strong-motion earthquake records with the electric analog computer," *Bull. seism. Soc. Amer.*, Jan. 1949, vol. 39, pp. 47-56.

This paper describes the use of an electric analog computer for the study of the effect of earthquake motions on single- and multiple-degree-of-freedom structures.

An electric-network analog of the structure is set up in the machine and it is excited with a voltage similar in wave form to that of the ground motion recorded on accelerograms. The outputs show the deflections of the various members of the structure. By this method a time saving of about 500:1 is achieved over numerical solutions for single-degree-of-freedom systems. Multiple-degree-of-freedom structures are also analyzed, for which the numerical solution time would be prohibitive.

The paper deals principally with the method. Results so far have shown that the response of a structure is primarily in the first mode of vibration.

Herbert Harris, USA

1351. M. Dumas and P. Maheu, "Statistical methods and their applications in the field of technical industries" (in French), *Mémor. Artill. fr.*, 1948, vol. 22, no. 86, pp. 837-967.

This is the second part of a treatise on statistical methods and their technical applications (see Rev. 396, Mar. 1949). In this part the authors discuss conventional topics in elementary statistics in a confused and unclear fashion. The reviewer does not believe that a person interested in technical applications will find this treatment particularly rewarding. A good portion of the text is devoted to graphical methods of analysis and smoothing of data. The reviewer is at a loss to see the virtues claimed for these methods.

Benjamin Epstein, USA

1352. Kurt Rantsch, "The optics of precision measurements" (in German), Carl Hanser Verlag, Munich, 1949. Paper, 9.75 × 6.75 in., 317 pp., 520 figs.

This is volume 2 of a series on technical measurements (*Tech-*

nisches Messen in Einzeldarstellungen). The basic geometric and physical optics is given (71 pp.), followed by 40 pages on lens systems. There are 141 pages on the basic optical instruments, on microscopic measuring devices and on optical scanners. A last chapter deals with methods which "put the specimen in the paths of the light rays" (interference devices, surface testers, plane-optical-beam methods, total reflection methods). Lists of notations, formulas and references are given at the end of the book. Ed.

1353. André Martinot-Lagarde, "Dimensional analysis—applications to fluid mechanics" (in French), *Off. nat. Étud. Rech. aéro. tech. Note*, no. 41, 1948, 180 pp.

The author treats dimensional analysis and the principle of similitude in six chapters, covering quantity, derived quantity, invariance, the fundamental theorem of Vaschy, similitude, and applications in fluid mechanics, respectively.

There are presented the usual examples of towing a ship's model, testing a submarine model, flow in pipes, as well as additional interesting cases including a study of the Kármán track, rotating solids, and compressible viscous flow.

A short bibliography of 60 references is provided. It seems that this might profitably have been made much more extensive in conformity with the literature. A good index and several short appendices are added.

The typical Gallic elegance of development of the work of the author, and his rather pointed reference to the debt due Reech and Vaschy, somewhat belie the actual history of the subject. This point is not controverted by the casual mention of Froude, Reynolds, Rayleigh, and Bridgman in the introduction.

It is the opinion of the reviewer that the systematic and logical presentation of the subject together with the many specific applications in the mechanics of fluid flow make the work a desirable reference.

William H. Hoppmann II, USA

## Mechanics (Dynamics, Statics, Kinematics)

1354. Th. de Donder and Paul Melchior, "Gauss' principle of least constraint applied to the dynamics of solid bodies with non-holonomic restraints" (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 12, pp. 966-968.

Th. de Donder and his collaborators have demonstrated in papers previously reviewed (see Revs. 1235, 1236, 1237, 1238, Oct. 1949, and Rev. 1346 in this issue) that the method of Lagrangian multipliers may be used, in conjunction with Hamilton's principle, to obtain the equations of motion of both holonomic and nonholonomic systems. In the present paper the authors show that the method can be used also with Gauss' principle of least constraint. The theory is illustrated by the nonholonomic problem of a hoop rolling on a plane. P. C. Dunne, England

1355. M. Casarini, "On the conical pendulum of variable length" (in Italian), *Boll. Un. mat. ital.*, Dec. 1948, ser. 3, vol. 3, pp. 251-255.

The author takes issue with the solution of an exercise in *Engineering Mechanics* by Timoshenko and Young [McGraw-Hill Book Co., 1937, p. 156] where it is assumed that a conical pendulum remains conical when its length is slowly changed. He shows that, for small oscillations about a vertical, and a prescribed, uniformly accelerated variation of the pendulum length, exact integration yields an elliptic trajectory of the pendulum bob.

However, if adiabatic (rather than exact) integrals are used, as derived by D. Graffi [*R. C. Accad. Lincei*, 1932, vol. 15, p. 657; *Ann. Math.*, 1936, vol. 15], the assumption discussed turns out to

be true for an arbitrary adiabatic variation of the pendulum length. A. W. Wundheiler, USA

1356. I. I. Artobolevski and B. M. Abramov, "Solution of the equation of motion of a machine for forces depending on velocities" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Mar. 1948, vol. 59, no. 9, pp. 1541-1544.

A method of approximations requiring the plotting of a number of graphs to determine the time variation of the angular position of a linkage is given. No mention of a machine is made.

John M. Kopper, USA

1357. F. M. Dimentberg, "An analogy between finite motions of plane and spatial four-bar linkages" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekhn. Nauk)*, Feb. 1949, no. 2, pp. 181-185.

A spatial four-bar linkage  $L$  (that is, one with nonparallel hinges), whose adjacent bars enclose, consecutively, the angles  $\phi$ ,  $\theta$ ,  $\chi$  and  $\psi$ , can be matched with a plane four-bar linkage  $L'$  so that, if  $\phi'$ ,  $\theta'$ ,  $\chi'$  and  $\psi'$  denote the corresponding angles of  $L'$ , the relation  $\phi = \phi'$  enforces

$$\chi = \chi', \xi \tan \psi'/2 = \tan \psi/2, \eta \tan \theta'/2 = \tan \theta/2$$

where  $\xi$  and  $\eta$  are determined by the dimensions of  $L$ . The dimensions of  $L'$  are explicitly expressed in terms of those of  $L$ ; only trigonometry (plane and spherical) is involved. The noteworthy fact is, of course, the existence of these simple formulas, rather than the mapping itself. A. W. Wundheiler, USA

## Gyroscopics, Governors, Servos

1358. Z. Blokh, "Contribution to the theory of servomotors for hydraulic systems" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekhn. Nauk)*, Jan. 1947, vol. 11, pp. 151-158.

Theoretical expressions are derived for the displacement, velocity and acceleration of an externally-loaded hydraulic actuating piston when the fluid admission port is suddenly opened fully. Bernoulli's equation for incompressible flow, the equation of continuity and Newton's law of motion are used. Pressure losses in the system are represented by Weisbach's formula and are included in an over-all "hydraulic transmission constant." The author recommends an experimental determination of this constant by measuring the piston displacement for a given interval of time upon sudden full-opening of the admission port.

When such an actuator is used as a power boost, a feed-back link is provided to close the admission port by means of the piston movement. Flow then occurs through a varying port opening. The author derives an expression for piston velocity vs. displacement for this case, approximating Weisbach's data for losses in partially closed tubes by means of an analytical expression. He concludes from this analysis that the piston comes to a stop without shock. Walter W. Soroka, USA

1359. Hubert M. James, Nathaniel B. Nichols and Ralph S. Phillips, "Theory of servomechanisms," *M.I.T. Rad. Lab. Ser.*, McGraw-Hill Book Co., New York, 1947. Cloth, 9.25 × 6 in., 375 pp., 159 figs.

This volume of the familiar series is the work of a group concerned principally with servomechanisms used in automatic-tracking radar systems. However, considerable design information applicable to other types of servos is presented as well. The

treatment presupposes considerable mathematical preparation. In many aspects of the development, a previous knowledge of servomechanisms is also helpful. To one of adequate background a careful reading is most rewarding.

The first chapter of the book is introductory. The next chapter summarizes the mathematical fundamentals of the presentation to follow, and among the subjects discussed are transformations, mapping, and stability criteria. Chapter 3 on servo elements describes the operation and characteristics of a number of servo components, and does so in excellent detail. General design principles for both single- and multiple-loop servomechanisms are presented in the following chapter, discussing equations and design criteria based on steady-state responses to sinusoidal disturbances. A worth-while addition to the chapter is a comprehensive treatment of two numerical examples illustrating the techniques described. Almost entirely mathematical, chapter 5 is concerned with filters and servos which are supplied with pulsed, rather than continuous, data.

The final three chapters approach the problem of servo design from a statistical point of view. Starting with an excellent phrasing of the problem of practical servo design, this section of the book then mathematically states servo properties and performance in the presence of noise. Emphasized is servo design based on the actual disturbances to be encountered. After a discussion of the theory of probability and statistics applied to servomechanism design, the minimum root-mean-square error criterion is presented and applied to typical examples in fire control.

As the above summary discloses, the volume presents many aspects of servo design appearing in no other book. Its emphasis on the importance of consideration of noise and the actual disturbances in servo design is noteworthy, although the reduction of the mathematical presentation to a particular problem may present formidable difficulties to the practicing engineer.

Most of the weaknesses of the book are the result of the circumstances under which it was written. It was prepared under a stringent time schedule, and portions of the book reflect haste in writing. If the effort of the authors had been directed toward gathering information for a book instead of solving urgent problems on military equipment, the book would undoubtedly be more complete and better integrated. Incidentally, all the useful information on servomechanisms in the Radiation Laboratory Series does not appear in this book, and those interested in the subject should read portions of volumes 17 and 21 as well.

It is felt that servo literature has been considerably enriched by the publication of this work, and it is well recommended as a worth-while advance study in the field.

William R. Ahrendt, USA

## Vibrations, Balancing

(See also Revs. 1350, 1385, 1395, 1396, 1458)

1360. H. Blume, "Theory and practice of periodogram analysis of recording instrument curves of the form of nonpersistent vibrations" (in German), *Z. angew. Math. Mech.*, July 1947, vol. 25/27, pp. 113-118.

(The author refers to his book, *Grundlagen und Methoden der Periodenforschung*, Berlin 1937, for the definition of "periodogram analysis" and "persistent vibrations." This book is not accessible to the editor, but it can be guessed that the first term denotes a method of analysis ignoring the phase but utilizing the amplitude-frequency dependence derived from only a portion of the vibration record, and estimating the involved error. As to the second term, its meaning is not relevantly involved in the paper reviewed. Ed.)

The author presents the mathematical foundations of periodogram analysis for the case that the record consists of vibrations of constant frequency  $\alpha$  and phase but of arbitrarily varying amplitudes. It is assumed that the range of the curve is much greater than the chosen interval of analysis (its left end point will be denoted by  $g$ ).

First the case of amplitude linearly varying with time is considered. It is shown that the frequency  $\alpha$  may be determined from the (roughly constant) slopes of the  $g$  vs.  $\psi$  plot for the component vibrations in the neighborhood of  $\alpha$ ,  $\psi$  being the measured phase of the component. Analysis of a curve with exponentially varying amplitudes leads to the same results.

Finally, the more general case of a curve with constant frequency but with arbitrarily varying amplitudes,  $y = f(t) \cos(\alpha t - \beta)$ , is treated. It is shown that relatively accurate values of the frequency  $\alpha$  may be obtained if the arithmetic mean of the amplitude  $f(t)$  changes only slightly with a displacement of  $g$ , i.e., of the interval of analysis. The amplitude itself may show considerable variation.

The author points out that periodogram analysis is of special value in the problem of vibration prevention, because in this case the important things are the frequencies, while the mere knowledge of the mean values of the amplitudes is sufficient to judge the importance of the single frequencies.

Heinz Parkus, Austria

1361. H. Pailloux, "Small periodic vertical motions of a flexible cable with finite weight and fixed ends" (in French), *C. R. Acad. Sci., Paris*, Apr. 19, 1948, vol. 226, pp. 1242-1244.

The general equations of motion for this case are enunciated and the problem is reduced, for the case of small motions, to the solution of a single ordinary differential equation. Frequency equations are given for the special cases in which the tension in the cable is large.

W. S. Hemp, England

1362. Robert Campbell, "On the vibration of an elliptic loudspeaker" (in French), *C. R. Acad. Sci. Paris*, Mar. 21, 1949, vol. 228, pp. 970-972.

In the investigation of the vibrations of a loud-speaker regarded as a plane elliptic plate, a Lamé differential equation containing three terms is obtained. The eigenvalues of the equation have been given by Mögliche [*Ann. Phys.*, 1927, vol. 83, p. 609], and in this paper the author gives an elegant alternative method for evaluating these values, based on the method developed by Ince [*Proc. roy. Soc. Edinb.*, 1939, vol. 60, p. 83] for the integration of the Lamé equation and on L. Brillouin's recent results [*Quart. appl. Math.*, 1948, p. 167] on the calculation of the characteristic exponent. The author's method has the advantage that the periodic solution of the equation is obtained in the form of a trigonometric series, rather than in the form of a series of powers of elliptic functions.

R. M. Davies, Wales

1363. M. Y. Leonov, "Parametric representation of quasiharmonic vibrations" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Sept. 11, 1948, vol. 62, pp. 161-162.

A single-degree-of-freedom oscillatory system is considered in which the damping  $\gamma(\theta)$  is a function of time  $\theta$ . By means of the transformation  $\gamma d\theta = \mu(\phi) d\phi$ , the damping  $\gamma(\theta)$  is expressed in terms of the parameter  $\phi$ . The function  $\mu(\phi)$  is assumed as a trigonometric polynomial, and it is indicated under what values of the coefficients the solutions are stable or unstable in accordance with Liapunov.

Walter W. Soroka, USA

1364. N. S. Nagendra Nath and Sanat Kumar Roy, "The vibrations of an infinite linear lattice consisting of two types of particles" (in English), *Proc. Indian Acad. Sci. Sect. A*, Nov. 1948, vol. 28, pp. 289-295.

This solution of the problem of the vibration of two types of particles, arranged alternately in a lattice, brings forth the work of Lagrange, Hamilton (for a finite lattice, rather than infinite as the authors imply), and Riemann. The normal vibrations of a lattice are given according to C. V. Raman. Starting with the general assumption that forces acting on a particle are proportional to changes in the position of all the particles along the line, and that initially one particle is slightly displaced along the lattice, a later and simplifying assumption is resorted to, that forces acting on a particle arise only due to changes in the neighboring distances.

As mentioned in one of Hamilton's problems, for transverse vibration, there is generally a system of  $n$  simultaneous, differential equations of the second order to be integrated. In the present case there is the laborious task of solving an infinite number of second-order differential equations—each of which consists of an infinite number of terms. Certain boundary conditions can make the solution practically impossible; the authors have chosen suitable boundary conditions for facilitating the solution. In order to find the nature of the solution, the Riemann surface is used; the method of steepest descent is applied, to find the asymptotic nature of the solution.

For engineering use, an explanation of the subscript notation, Eq. [1], would be desirable, together with some typical applications.

Eastman Smith, USA

## Wave Motion, Impact

(See also Revs. 1395, 1396, 1440)

1365. Albert E. Heins, "Water waves over a channel of finite depth with a dock," *Amer. J. Math.*, Oct. 1948, vol. 70, pp. 730-748.

This paper presents a mathematical treatment of surface waves in an infinite channel of finite depth with a semi-infinite rigid dock located on the upper surface of the channel. The mathematical problem is to satisfy Laplace's equation over an infinite slab of a finite width. The problem is reduced to a two-dimensional one and the appropriate Green's function is constructed. This leads to an integral equation of the Wiener-Hopf type with special assumptions about the behavior of the potential, which can be evaluated analytically. Solutions are given for the medium free of sources, and, in a second step, solutions for a logarithmic discontinuity at  $x = 0$ , that is for a source or a sink at the edge of the dock. No numerical evaluations are included, but they may be published later.

Horst Merbt, England

1366. Walter H. Munk, "Note on period increase of waves," *Bull. seism. Soc. Amer.*, Jan. 1949, vol. 39, pp. 41-45.

This note discusses various formulas worked out by the author and others for the increase in period  $T$  of an individual wave, the identity of which is conserved over a definite distance  $x$ . The wave may be in water or in the earth's surface and may be very near the traveling wave front. However, the formulas do not apply to period increase of ocean swell, and the author corrects an earlier statement of his in this connection. The Airy integral is applied to Love waves yielding the formula

$$T = 4.6 (312)^{1/2} V_0^{-1} (h\mu_1/\mu_0)^{2/3} (V_0^2/V_1^2 - 1)^{2/3} x^{1/2},$$

for the case of an infinitely thick lower layer. This formula may offer a means for determining the thickness  $h$  of an upper layer if

the velocities  $V_0$ ,  $V_1$  of the shear waves and the rigidities  $\mu_0$ ,  $\mu_1$  of the two layers are known.

W. C. Johnson, Jr., USA

## Elasticity Theory

(See also Revs. 1375, 1386)

1367. M. T. Huber, "Theory of elasticity, Vol. I" (in Polish), Polish Acad. Sci., Kraków, 1948. Paper, 7 × 10 in., xv + 387 pp., 82 figs.

This book contains much new material of research value. The first two chapters presenting a general discussion of strain and stress are followed by a chapter concerned with stress-strain relations and the general equations of elasticity. The author gives his own derivation of the principle of superposition and of the criteria of elastic failure, with special regard to the maximum-strain-energy theory, proposed by the author in 1904 and treated independently by R. von Mises (1913) and H. Hencky (1923). The following three chapters deal with two-dimensional problems, using rectangular and polar coordinates and complex variables. In the sixth chapter the treatment of riveted junctions by A. Grzedzielski is presented, as well as a simple proposal of W. Olszak to use the "generalized Poisson's ratio" for a formally unified treatment of plane stress and strain. The seventh chapter deals with the general energy methods.

In the eighth chapter, dealing with concentrated loads, the author presents the detailed solutions of the equations of Hertz's theory in two cases: (1) when the area of contact is circular (e.g., two spheres in compression), and (2) when the area forms a strip of constant width (two cylinders with parallel axes). The corresponding diagrams of stress lines are given. The author completes here his argument of 1904 and explains the failure of Hertz's theory of hardness.

The ninth chapter deals with some of the most important three-dimensional problems. Special attention should be given to the tenth chapter in which bending and torsion of beams are treated. In this chapter the concepts of center of twist and center of shear are analyzed in a most detailed and precise manner. General formulas for the coordinates of these centers are given. The author shows that the theorem of C. Weber (quoted in A. and L. Föppl's *Drang und Zwang*, vol. II) stating the identity of these points is not, in general, true.

The first volume ends with a discussion of energy methods and mathematical analogies. A study of beams of elliptic cross section is used to correct Föppl's opinion (see above) that the strength of a nonround beam in torsion is unaffected by constraints preventing the warping of one or more cross sections.

With regard to the volume of its subject matter, the book can be classified between those of Timoshenko and Love; with regard to its rigor, it is their match.

J. Nowiński, Poland

1368. Samuel Levy and Frank C. Smith, "Stress distribution near reinforced circular hole loaded by pin," *J. Res. nat. Bur. Stands.*, Apr. 1949, vol. 42, pp. 397-404.

This paper contains an approximate solution of an important practical problem. The problem is that of a flat plate with a reinforced circular hole. The boundary of the hole is loaded by means of a rigid pin and the hole is not permitted to distort under load. The solution is obtained from a plane-stress theory. The problems of the reinforcing ring and the unreinforced plate are first solved separately. These solutions are then joined by matching displacements and forces at the common boundary. The boundaries of the plate which are parallel to the direction of the load are not stress-free. This inconsistency is not of primary importance and does not appreciably affect the maximum stresses.

The theoretical results were experimentally verified and the comparison of test and theory is summarized in the paper.

Paul F. Chenea, USA

1369. A. Pflüger, "The semi-infinite plate with edge reinforcement. A stress analogy to the problem of the lifting line" (in German), *Z. angew. Math. Mech.*, Oct. 1947, vol. 25/27, pp. 177-185.

The author deals with the following problem: A semi-infinite plate is stressed in its plane by tensile forces acting parallel to the edge. Fixed to this edge is a reinforcement of finite or infinite length with arbitrarily varying cross section.

Under the assumption that the reinforcement acts as a bar of negligible flexural stiffness, only shearing forces can be transmitted between the two bodies. For the stress in the reinforcement an integrodifferential equation is then deduced which formally agrees with Prandtl's famous equation for the spanwise lift distribution of a wing. The induced velocity corresponds to the addition stresses in the plate.

It is shown that in general the normal stresses as well as the shearing stresses in the plate tend to infinity at both ends of the reinforcement, even if the thickness of the reinforcement and its rate of increase are zero at these points. A necessary and sufficient condition for finite stresses is given.

Heinz Parkus, Austria

1370. D. N. Mitra, "On the flexure problem for some boundaries" (in English), *Bull. Calcutta math. Soc.*, Dec. 1948, vol. 40, pp. 173-182.

This paper deals with the problem of flexure of an isotropic elastic cylinder when the load acts through the center of flexure of the end of the beam. The function-theoretic method has been used to find solutions for three boundaries: inverse of an ellipse with respect to its center, two orthogonal circular arcs of equal radii, and a loop of the lemniscate of Bernoulli.

R. M. Wingren, USA

## Experimental Stress Analysis

1371. Hugh Ford, "Mechanical methods for the measurement of internal stress," *Inst. Metals Monogr. Rep. Ser.*, no. 5, 1948, pp. 3-11.

A review of the experimental methods available for determining internal stresses in cylinders, tubes, flat plates, and bars is given, along with the appropriate formulas for reduction of the experimental data. A bibliography of significant literature on the subject is also included in the paper.

A. H. Flax, USA

## Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1370, 1385, 1398)

1372. R. W. Hofmann, "The efficiency of plate girders and their most economical height" (in German), *Bauplan. Bautech.*, Oct. 1948, vol. 2, pp. 275-282.

In the first part of the paper the author discusses the minimization of the ratio of the integral, taken over the length, of the section modulus, to the volume of a girder. He takes into account such factors as minimum allowable thickness of web, and includes in the volume such elements as stiffeners, butt straps, rivet heads, and excess length of cover plates. Numerical examples are given.

In the second part of the paper the relations between moment

of inertia of the cross section, the cross-sectional area, and the depth of a girder are discussed.

Finally the most economical design of a girder is considered under given conditions of maximum allowable stress and maximum allowable deflection.

Wm. R. Osgood, USA

1373. R. William Höjlund, "Calculation of bending of beams with constant moment of inertia" (in Swedish), *Tekn. Tidskr.*, Mar. 26, 1949, vol. 79, pp. 237-240.

The author shows how the influence lines for deflections can be used, by integration of the elements of area, for the drawing of a diagram from which the deflections for beams with uniformly distributed load can be read. Another diagram is presented for beams with triangular distributed load. The diagrams are also useful for the determination of reactions in continuous beams.

A. R. Holm, Denmark

1374. N. O. Myklestad, "A tabular method of calculating helicopter blade deflections and moments," *J. appl. Mech.*, June 1948, vol. 15, pp. 97-106.

This paper presents a method of tabular integration for the beam-bending equation of a rotating helicopter blade. Only the case of bending out of the plane of rotation is considered, and one of the principal axes of the blade is assumed parallel to this axis.

Both constant and periodically varying loads are treated, the latter by decomposition into constituent harmonic terms. The analysis for constant loads is similar to the one currently in use in helicopter-blade analysis, first proposed by Stuart [*J. aero. Sci.*, Apr. 1943], and improved by the author [*J. aero. Sci.*, Oct. 1943]. The analysis for harmonically varying loads is based on the assumption that the loads may be computed from the usual helicopter theories as if the blade were rigid, and subjected to certain rigid body motions—such as harmonic feathering—or harmonic oscillation of an aileron. These loads are then applied to the rotating blade, which is considered at this stage to be flexible—but not subjected to the motions and aerodynamic actions which produced the original loads. The additional inertia, centrifugal, and aerodynamic forces due to the deflection of the blade are computed and superposed on the original loading to obtain the final blade loading.

The aerodynamic forces due to blade deflection are computed on the basis of strip theory, using the aerodynamic forces on a harmonically oscillating two-dimensional airfoil, as in flutter theory. All forces and deflections are expressed in complex form. The tabular integration is carried out by dividing the blade into sections and assuming that loads and masses are concentrated at the ends of sections only, so that the shear and centrifugal force are constant along a section. A tip slope and deflection are assumed as initial conditions for the tabular integration; these are determined (after the integration has been carried in to the blade root) from the boundary conditions at the root. An appendix indicates how blade torsional deformation may be included in the analysis.

A. H. Flax, USA

## Plates, Disks, Shells, Membranes

(See also Revs. 1371, 1377, 1383, 1389)

1375. S. S. Manson, "Determination of elastic stresses in gas-turbine disks," *Nat. adv. Comm. Aero. Rep.*, no. 871, 1947 (issued in 1949), 11 pp.

In this report the author presents a finite difference solution for the stresses in gas-turbine disks. The method assumes a two-dimensional state of stress. An allowance for radial variations in

thickness, temperature, modulus of elasticity, coefficient of thermal expansion, material density and Poisson's ratio is included in the solution. The procedure is direct and is so arranged that the necessary calculations may be made by nontechnical personnel. Illustrative examples are included. Paul F. Chenea, USA

## Buckling Problems

**1376. Georg Schnadel, "The strength of transversely stiffened decks of ships,"** *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 256-267.

The hull of a ship is generally considered to be a beam bent by its own weight, by its loading, and by the action of the surrounding water. The strength of the hull in bending depends essentially on the buckling strength of the flange plates. Of special importance is the deck, which has only relatively light reinforcement and is usually transversely stiffened in merchant-ship building. The buckling problem of the hull is formulated in this paper as follows: Consider the deck to be an infinitely long plate with transverse stiffeners. One of the long edges of the plate is assumed to be simply supported and the other edge is considered to be jointed to another transversely stiffened plate. The second plate may be either simply supported or elastically built-in along its other long edge. The compressive stress in the deck plate is assumed to be uniformly distributed and the compressive stress in the side plate to decrease linearly. The approximate magnitude of the buckling load is determined by the energy method.

C. T. Wang, USA

**1377. Herman F. Michielsen, "The behavior of thin cylindrical shells after buckling under axial compression,"** *J. aero. Sci.*, Dec. 1948, vol. 15, pp. 738-744.

The postbuckling behavior of thin cylindrical shells under axial compression is studied by an extension of the investigations of von Kármán and Tsien. The minimum stress sustained after buckling is found to be  $0.195Et/R$ . A maximum stress of  $0.77Et/R$  is also reported, but it is considered questionable by the author due to the approximation assumed in the deflection surface.

Conrad C. Wan, USA

**1378. P. P. Bijlaard, "Fundamental observations regarding the buckling of plates and shells in the plastic range"** (in German), *Mitt. Inst. Baustat. Eidgenöss. tech. Hoch. Zürich*, no. 21, 1948, 32 pp.

This rather discursive paper is one of the few which have been written so far on the plastic buckling of plates and shells. Stress-strain relations and energy of deformation are considered in some detail. The author compares the plastic relations, which he develops, to the anisotropic stress-strain relations for crystals.

With the aid of the plastic stress-strain relations a differential equation for the plastic buckling of plates, similar in form to that for elastic buckling, is developed. In a table is presented a comparison of the number of half-waves into which a plate buckles, calculated from this theory, and the number given in published experiments. In another table is given a comparison of the buckling stresses found by the author, with the buckling stresses predicted by elastic theory, for 12 plates with various edge conditions.

No equations for the plastic buckling of shells are given. The title of the paper apparently refers to some general discussion of the earth's crust as a shell "floating" on a substratum. Some very interesting results of the application of a plastic theory of local deformation to explain the formation of continents and

archipelagoes are given in the form of maps with superimposed calculated networks giving the directions of "lines of flow."

W. H. Hoppmann II, USA

## Structures

(See also Revs. 1376, 1460, 1461, 1470)

**1379. M. D. Brisby, "Vierendeel girder analysis,"** *Engineering*, Jan. 14, 1949, vol. 167, pp. 25-27.

The Vierendeel girder is considered to be a system of continuous frames. For each frame 4 four-moment equations can be written, whose number is thus equal to the number of unknown boom end-moments. The post end-moments are then determined by the equations of equilibrium of moments at the joints. Since the four-moment equations involve the angles of drop, another equation is needed for each panel; this is obtained from the condition of equilibrium between the boom moments and the vertical shear forces through the panels. For  $m$  panels there are available  $5m - 1$  equations for  $4m$  unknown moments and the  $m - 1$  unknown deflections. This system of equations is solved by the Gauss method of successive elimination. A numerical example is given.

F. Stüssi, Switzerland

**1380. A. van der Neut and F. Plantema, "Stress distribution in wings with two nonparallel spars connected by elastically deformable ribs and skin"** (in Dutch), *Nat. LuchtLab. Amsterdam Rap.*, 1947, no. S. 251, 18 pp.; no. S. 279, 9 pp.; no. S. 326, 3 pp.

The authors consider two spars connected by parallel ribs. The rib system is schematized, each rib considered replacing a group of actual ribs, its shear rigidity being assumed equal to the total shear rigidity of the group. Ribs are assumed at all points where there are discontinuities, as well as at other more or less arbitrarily chosen points. Between the ribs the spar flanges and the lines through the torsional centers of the spars are straight while at the ribs their directions may change.

In report S. 251 an exact consideration of the torsional rigidity of the spars was omitted, while in report S. 279 the theory is extended for the case of spars with a finite torsional rigidity by the assumption of suitable torsion tubes.

The skin is connected with the flanges and with the ribs and is supported by a continuous system of stiffeners running parallel to the ribs. The loads are applied normally to the plane of the wing and are assumed to be symmetric or antisymmetric.

From the theorem of least work a recurrent system of linear algebraic equations is derived which is sufficient to determine the bending moments in the spars and the torsional moments in the torsion tubes. The solution of this recurrent system is discussed. In reports S. 251 and S. 279 the solution is found by the well-known method of superposition of the nonhomogeneous equations, starting with arbitrary values for a suitable number of unknowns and mutually independent solutions of the homogeneous equations so that the boundary conditions are satisfied.

In report S. 326 it is stated that, because of difficulties experienced in a numerical example in getting sufficiently accurate results with the above method of solution, several other procedures were tried. It is concluded that the Gaussian method of solving a set of simultaneous linear equations seems to be the most adequate. In this report the authors also consider the particular case of wings with the skin partly absent.

W. L. Esmeijer, Holland

1381. P. Csonka, "Analysis of frames with movable joints" (in English), *Publ. tech. Univ. Budapest (Műegyetemi Kozl.)*, 1948, no. 2, pp. 22-36.

The method presented is a modification of the Hardy Cross method of analysis of frames and gives a better convergence, especially when the beams are rigid in comparison to the columns. The Cross method has two groups of partial deformations, pure rotation and pure translation. The author adds a third group, pure distortion. Using this the author considers a panel consisting of one horizontal beam and the adjoining vertical columns. The deformation of the panel is characterized by the horizontal displacement of the two adjoining stories with respect to the beam considered, and by the rotation of the joints of the beam, which have to rotate through the same angle  $\varphi$ . The three conditions—necessary to calculate the two displacements and the angle  $\varphi$  are—that the sum of the horizontal forces in each adjoining set of columns is zero, and that the sum of the resulting moments in the joints of the horizontal beam is zero. Pure distortion is then applied to one after the other of the panels into which the whole frame can be decomposed.

J. J. Koch, Holland

1382. Radu Prişcu, "Contribution to the study of lines of equal principal stresses in a vertical faced gravity dam" (in French), *Bul. Polit. Bucarest*, May-Dec., 1948, vol. 17, pp. 130-144.

The author develops equations for the variations of the principal and maximum shearing stresses in a gravity dam of triangular cross section with a vertical upstream face. Presumably the water is assumed to be even with the top of the dam. The development is based on expressions for plane stress distribution attributed to Ludin-Tölke [*Wasserkraftanlagen*, vol. 9] and D. Pavel [*Căderi de apă*, vol. 1]. In these expressions the normal stresses are indicated as being functions of the porosity and water-cement ratio of the concrete, in addition to the usual geometric and specific weight factors. Glenn Murphy, USA

1383. Åke Holmberg, "Calculating flat slabs of reinforced concrete" (in English with French and German summaries), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, prelim. publ., pp. 499-506.

The author gives recommendations and methods for the design of flat building slabs, based in part on tests of six circular plates with, however, only a ring support instead of a column capital at the center. He recommends the use of radial and circumferential bars in general. However, practical considerations usually make a rectangular arrangement of reinforcement more desirable; the greater toughness of such an arrangement seems to have been inadequately considered in the tests and in the design recommendations.

N. M. Newmark, USA

1384. Henry Lossier, "Self-stressing concrete by using expansive cements" (in French), *Mémo. Soc. Ing. civ. Fr.*, Mar.-Apr. 1948, vol. 101, pp. 189-225.

Prestressing obtained by stretching the steel reinforcing bars or wires independently of the concrete, either before the concrete is poured (pretension) or after it is set (posttension), is one of the most important developments in concrete technique in the last 20 years. Both methods, however, require special equipment to stretch the steel.

The author of this paper introduces a new and important change in these techniques by developing an expansive cement. During setting the cement, instead of shrinking, expands and puts itself in compression by stretching the steel. The cement

used is a mixture of Portland, a sulfoaluminum cement and a stabilizer. The author claims that with the right mixture of the three elements it is possible to obtain any desired expansion in any desired time. Mortars made with expansive cements have the same strength as the base cement at 28 days, and progressively higher strength after that age. The paper includes the results of several long-time tests made with the new cement, and a description of many applications, mainly in the fields of foundations, bridges and highways.

A. J. Durelli, USA

1385. W. Oppelt, K. Kammüller, K. Karas, W. Swida, and G. Denkhäus, "Elasticity and plasticity" (in German), *Z. angew. Math. Mech.*, Aug.-Sept. 1947, vol. 25/27, pp. 165-170.

These are summaries of five lectures delivered at the Deutsche Mathematiker-Tagung (German Mathematicians Conference), Karlsruhe, 1947. Oppelt's lecture reviews a paper of O. Föppl ["Damping during alternating torsional loading of preloaded bars," *Ingen.-Arch.*, 1947, vol. 16, pp. 107-110]. It contains a proposal for the calculation of the relative damping, based on test results.

Kammüller's paper concerns the determination of the influence of changes in cross sections of statically indeterminate systems. It describes a procedure for recomputing the statically indeterminate quantities after making small changes in certain elasticity coefficients, which makes use of the originally determined values.

Karas reports on "Determination of the pressure exerted by freshly filled-in concrete on the surrounding shell," taking account of hardening and friction. An approximate method is indicated, the results of which agree reasonably well with those of exact calculations.

Swida describes a "Procedure for determining structural deflections in the elastoplastic range," his paper dealing with straight and curved beams in bending.

Finally, Denkhäus reviews another paper of O. Föppl, published in *Metalloberfläche*, 1947. This is concerned with surface rolling of cylindrical torsion-rod springs so as to obtain optimum fatigue properties. The results of tests do not agree with Hertz' equations in that no simple rules can be established for the maximum residual stresses and the thickness of the surface layer where compressive stresses occur.

F. J. Plantema, Holland

## Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1391, 1393, 1397, 1399, 1400, 1401, 1404, 1405, 1470)

1386. I. I. Goldenblat, "On a method in the theory of elastic and plastic deformations" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Aug. 21, 1948, vol. 61, pp. 1001-1004.

The author claims that the basic thermodynamic relations for reversible isothermal deformations of a solid may also be applied to (irreversible) plastic deformations, as long as unloading is avoided. Assuming the free energy to be a function of the linear and quadratic invariants of the tensors of stress and strain, the author obtains stress-strain relations for plastic materials with or without strain hardening.

W. Prager, USA

1387. P. Hodge and W. Prager, "A variational principle for plastic materials with strain-hardening," *J. Math. Phys.*, Apr. 1948, vol. 27, pp. 1-10.

After introducing a suitable definition of loading and unloading of a plastic material with strain hardening, the authors establish the following variational principle. The volume integral of

$\dot{\sigma}_{ij}\dot{\epsilon}_{ij}$  over the entire body is less for the actual stress rates than it is for any system of artificial stress rates which satisfy equilibrium and boundary conditions, and which either are indefinitely near the actual stress rates, or constitute unloading in all regions of the body where the actual stress rates constitute unloading. (Here  $\dot{\sigma}_{ij}$  and  $\dot{\epsilon}_{ij}$  represent the rates of stress and strain.) An extension to certain mixed boundary value problems is also indicated. The results apply only to cases when loading from the stress-free natural state is followed by at most one unloading.  
*Courtesy of Mathematical Reviews* F. B. Hildebrand, USA

1388. G. Colonnetti, "A general approach to the problem of viscous deformation" (in Italian), *R. C. Accad. Lincei*, May 1948, vol. 4, pp. 515-519.

The author develops an equation defining the variation of state, in terms of stress and deformation, from the beginning of viscous flow in a homogeneous medium subjected to a given system of forces and impressed deformation. The question developed for viscous deformation is similar in form to the one for plastic deformation. Discussion of the application to the evaluation of strains in a homogeneous solid subjected to viscous deformations proportional to the corresponding stresses is included.

Glenn Murphy, USA

1389. A. Nádai, "Theory of the plastic distortion of thick-walled cylinders," *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 430-448.

The author determines strains and stresses in nonviscous plastic flow of thick-walled hollow cylinders loaded by a longitudinal force combined with an inner or outer radial pressure. For small radial, tangential and axial strains, the well-known relations

$$\epsilon_r = -\epsilon_0/2 - c/r^2, \quad \epsilon_t = -\epsilon_0/2 + c/r^2 \quad (1)$$

hold, while the stress-strain dependence is given by

$$\epsilon_r = \varphi[\sigma_r - (\sigma_t + \sigma_z)/2], \text{ etc.} \quad (2)$$

For finite strains defined by

$$\epsilon_r^* = \ln(1 + \epsilon_r), \quad \epsilon_t^* = \ln(1 + \epsilon_t), \quad \epsilon_z^* = \epsilon_0^* = \ln(1 + \epsilon_z),$$

the relations (with  $u = d\epsilon^*/dt$ )

$$u_r = -u_0/2 - c/r^2, \quad u_t = -u_0/2 + c/r^2, \quad u_z = \psi[\sigma_r - (\sigma_t + \sigma_z)/2], \text{ etc.}$$

replace Equations (1) and (2),  $u_r, u_t, u_0 = u_z$  being the strain velocities,  $c$  a constant and  $\psi$  the flow function that replaces  $\varphi$ . Since the form of Equations (1) and (2) is preserved for finite strains the same mathematical expressions will be obtained as for small deformations.

For nonviscous plastic flow at constant reference stress  $\sigma_0$ , where

$$2\sigma_0^2 = (\sigma_t - \sigma_r)^2 + (\sigma_z - \sigma_t)^2 + (\sigma_r - \sigma_z)^2,$$

and for small plastic strains, the following relations hold [with  $\sigma_1 = \sigma_0/\sin(\pi/3)$ ]:  $\sigma_r = -\frac{1}{2}\sigma_1 \ln \tan(\pi/4 + \alpha/2) + c_1$ ,  $\sigma_t = \sigma_r + \sigma_1 \sin \alpha$ ,  $\sigma_z = \sigma_r + \sigma_1 \sin(\frac{2}{3}\pi - \alpha)$ ;  $c_1$  is a constant of integration, determined from the boundary stresses, and  $\cot \alpha = (\epsilon_0 r^2/c)\sin(\pi/3)$ . These relations persist for finite deformations, as observed before. In this case one obtains for the flow pressure  $p$  the value  $\sigma_1 \ln(b'/a')$  where  $b'$  and  $a'$  are the deformed radii of the cylinder.

If strain hardening occurs, the hardening function being  $\tau = f(\gamma)$ ,  $\tau$  and  $\gamma$  being the octahedral shear stress and shear strain, respectively, the author proves that for pure radial flow

$$p = (3/2) \gamma_b \int \gamma^a \tau d\gamma / (r'^2/r^2 - 1)$$

where  $\gamma = \sqrt{(2/3)\ln(r'^2/r^2)}$ . If the hardening function is linear,  $\tau = \tau_0(1 + c\gamma)$ , plastic deformation sets in when  $p = \sigma_1 \ln(b/a)$ . For  $c < 1.225$  the flow pressure decreases, for  $c > 1.225$  it has a maximum in the course of the deformation. If axial flow occurs too, corresponding relations for  $\sigma_t$  and  $\sigma_z$  can be derived, and the author obtains an expression for the inner pressure.

E. Siebel, Germany

## Failure, Mechanics of Solid State

(See also Revs. 1385, 1393, 1409)

1390. P. G. Jones and W. J. Worley, "An experimental study of the influence of various factors on the mode of fracture of metals," *Proc. Amer. Soc. Test. Mat.*, 1948, vol. 48, pp. 648-663.

The factors related to the external conditions imposed on a given metal, which determine whether the mode of fracture is of a ductile or a brittle type, are considered to be rate of strain, stress concentration, state of stress and temperature. The paper presents experimental results on the influence of combinations of these factors on the mode of fracture of three steels and an aluminum alloy. The effect of strain aging on the mode of fracture of a semikilled steel and a rimmed steel was also investigated.

The stress concentration and state of stress were varied by making tests on notched and unnotched specimens in tension and on unnotched specimens in torsion. The rate of strain was varied by making static and impact tests in tension and in torsion. Test temperatures from room temperature to  $-310^\circ\text{F}$  were used in determining the temperature of transition from a ductile to a brittle type of fracture.

T. J. Dolan, USA

1391. Albert Kochendörfer, "Calculation of tensile strength of metals, and its dependence on the strain rate and temperature" (in German), *Metallforsch.*, June 1947, vol. 2, pp. 173-186.

According to an earlier paper of the author [*Reine angew. Metallk.*, 1941, vol. 7] the cubic metals (in contrast to the hexagonal ones) possess a physically defined proportional limit  $\sigma_0 = \sigma_0^* + \sigma_F$  where  $\sigma_0^*$  is the average of the proportional limits of a monocrystal over all orientations, and  $\sigma_F$  the stress hardening due to inhomogeneous deformation of the crystals. Beyond this limit the true stress  $\sigma_w$  equals  $\sigma_0 + \sigma_{wV}$ , the second (strain-dependent) term being the mean value (over all orientations) of the monocrystal-true-stress portion due to stress hardening. Denoting by  $\tau_V^*$  the hardening-caused portion of the monocrystal critical shear stress, the relation  $\tau_V^* = \sqrt{(p^*a)}$ , where  $a$  is the slip, is stated without explicit reference, and the reader is referred to the previously quoted paper for the relations  $\sigma_{wV}^* = 2.24 \tau_V^*$  and  $a = 2.24\delta$  ( $\delta$  is the strain). Hence

$$\sigma_{wV} = \sqrt{(p\delta)} \text{ with } p = (2.24)^2 p^*,$$

and a stress-strain law in the plastic range is obtained.

The proportional limit and the hardening parameter  $p^*$  of the polycrystal depend on the strain rate  $\omega = d\delta/dt$  and the absolute temperature  $T$ , both determined by the corresponding quantities for the monocrystal. These dependences are given in the author's earlier paper, below the temperature of fusion  $T_m$ , as

$$\sigma_0^* = \sigma_{00}^*[1 - \beta_1 \sqrt{(1 - \log \omega / \log \alpha)} \sqrt{T}]$$

with

$$\beta_1 = 0.034 \text{ and } \alpha = 4.8 \times 10^{12}.$$

The stress-hardening portion  $\sigma_F$  of the proportional limit, due

to lattice distortions, for moderate strain rates depends on  $T$  only. Below the recrystallization temperature  $T_R$  this dependence is

$$\sigma_F = \sigma_F' = \sigma_{F0} \sqrt{(1 - T/T_R)},$$

and above the temperature  $T_R$

$$\sigma_F = \sigma_F' (T_s - T)^2 / (T_s - T_R)^2.$$

The strain-rate and temperature dependence of the polycrystal parameter  $p$ , proportional to  $p^*$ , is given under normal conditions by

$$p = p_0(1 - T/T_s)[1 - B \exp(-D/\sqrt{T})],$$

$$B = B_1 \omega^{-\gamma}, D = D_1 / (1 - \log \omega / \log \alpha).$$

The constant  $\gamma$  can be derived from experiments of J. Weerts [*Forsch. Arb. Ver. deutsch. Ing.*, 1929, no. 323] for aluminum and is about 0.015. The formulas must be modified somewhat for large strain rates, since in this case the migration velocity of the dislocations cannot be considered infinite.

With the help of the relations derived, the strain-rate and temperature dependence of the tensile strength in aluminum and copper was calculated from the values for monocrystals, and was compared with the experimental results of Nádai and Manjoine [*J. appl. Mech.*, 1941, vol. 8, p. 77] at strain rates of  $10^{-2}$  to  $10^6$  cm per sec at various temperatures. Satisfactory agreement was obtained.

E. Siebel, Germany

## Design Factors, Meaning of Material Tests

(See Rev. 1390)

### Material Test Techniques

(See also Revs. 1385, 1390, 1391, 1406, 1407, 1408)

1392. B. Z. Sölyom, "Wear testing machine" (in English), *Publ. tech. Univ. Budapest (Műgytemi Közl.)*, 1948, no. 3, pp. 179-188.

The article contains a discussion of the physical variables involved in wear tests and a description of a testing machine designed to measure them. A few innovations are incorporated in the design, and results of several tests are given to show the quality of data obtainable.

George H. Lee, USA

1393. Erich Siebel and Siegfried Schwaigerer, "On the mechanics of the tension test" (in German), *Arch. Eisenhüttenw.*, 1948, vol. 19, pp. 145-152.

This is a compact analysis of the nature and consequences of necking in unnotched round bars. Part of it was published previously [*Wiss. Abh. dtsh. Materialprüf.*, 1944, ser. 2, no. 5, pp. 1-4]. The sections are:

1 *Distribution of stress.* By plausible assumptions and approximations, not all stated, the radial and axial principal stresses are calculated; hoop stress is taken equal to radial stress throughout (this assumption is discussed in an appendix). The trajectories of axial stress are assumed to have a profile radius  $\rho_x = \rho_r(r/x)^n$  where  $r$  is the sectional radius. With  $n$  unity, the peak axial stress and triaxiality at the center of the neck are somewhat higher than Bridgman's values. Flow stress  $k_f$  (axial minus radial) is assumed uniform in the minimum section.

2 *Flow curve.* The radial stress in the neck increases the load needed for flow. Evaluation of  $k_f$  requires observation of  $\rho_r$ . A series of profiles at six stages of flow is shown; except near the minimum section,  $\rho_r < 0$ , that is meridional profiles are convex out-

ward. Results of this analysis are given for four materials, conventional stress, flow stress, axial stress and peak stress being plotted against reduction in area  $\psi$ ; they increase in the order named. It is assumed that rupture begins at the axis and is quickly followed by separation. The peak axial stress at the highest  $\psi$  observed is called "technical cohesive strength."

3 *Necking process.* The inclination  $\alpha$  of the maximum principal stress trajectory is zero at the axis. Elsewhere  $\alpha$  enters along with  $\rho_r$  into the relations between the principal stresses and the load; these are reduced (without proof) to terms of a correction to the ratio of conventional stress to flow stress, based on the observed values of  $\alpha$  at the surface. These considerations are said to permit calculation of the profile of the neck from the flow curve found in a compression test. Observations of tensile necked profiles reveal a region about the minimum section where flow continues and the meridional profile is concave, separated by a discontinuous boundary (which progressively moves inward) from a region in which flow has ceased and the profile is convex.

4 *Significant data from tensile test.* The flow curve,  $k_f$  vs.  $\psi$ , characterizes the material and would be the same by any test. In principle, the successive forms taken by a tensile specimen could be calculated from the flow curve, and hence the ultimate strength and reduction in area. The cohesive strength at fracture is hard to measure in ordinary tests, but in any case is less significant than the cohesive strength in the unstrained condition, which must have a higher value. This last has not been measured, but it is said that it can be approximated by multiplying the ultimate tensile strength by the ratio of the areas of the original section to the fracture surface.

W. P. Roop, USA

1394. Forest Products Laboratory Staff, "Methods of tests for determining strength properties of core material for sandwich construction at normal temperatures," *For. Prod. Lab. Rep.*, no. 1555, March 1948, 19 pp.

The report describes the procedures which have proved satisfactory in the testing of strength properties of materials used for the cores of sandwich constructions. Methods of selection of specimens, control of moisture content and observation of response to loading are outlined for compression, tension, shear and torsion tests. Photographs of testing equipment are included.

A. M. Freudenthal, USA

1395. W. P. Mason, W. O. Baker, H. J. McSkimin, and J. H. Heiss, "Measurement of shear elasticity and viscosity of liquids at ultrasonic frequencies," *Phys. Rev.*, Mar. 15, 1949, vol. 75, pp. 936-946.

Two years ago, one of the authors [see Rev. 4, Jan. 1948] showed experimentally that, in addition to normal viscosity, liquids possess shear elasticity when the rate of stressing is sufficiently high. The elastic effect becomes significant at a relaxation frequency  $f_r$ , which is approximately proportional to the ratio of the shear elasticity  $\mu$  to the shear viscosity  $\eta$ . For light, mobile liquids such as water,  $f_r$  is extremely high and lies outside the limit of present-day techniques.

With long-chain-polymer liquids,  $\eta$  can vary between  $10$  and  $10^6$  poises and  $f_r$  between  $10^5$  and a few cps, and the existence of shear elasticity in these liquids was demonstrated by Mason from measurement of the resonant frequency and the damping of a quartz crystal set in torsional oscillation, first in air and then in the liquid. The disadvantage of this method lies in the fact that the size of the oscillating crystal becomes inconveniently small for frequencies exceeding  $5 \times 10^5$  cps.

To overcome this limitation a new method has been devised, based on the effect of a liquid on the reflection of shear stress

waves in a solid. To simplify the interpretation of the experiments, the stress waves are suitably polarized and they are generated in the form of pulses of short duration by a crystal and transduced, after reflection, by a second crystal. The pulses are set up in a rod of fused quartz, and a comparison of the reflected signal with and without a suitably-placed layer of liquid, enables the shear elasticity of the liquid to be determined. In this way the upper frequency limit has been raised to  $6 \times 10^7$  cps and experimental results for polyisobutylene and poly- $\alpha$ -methylstyrene liquids show that these liquids possess two relaxation frequencies, the first in the region of  $10^5$  cps and the second in the high-megacycle range; the shear elasticity associated with the latter is about 200 times as great as that associated with the former.

These results are interpreted in terms of the various types of changes which can occur in the configuration of the molecules, and it is worth noticing that the higher value of the shear elasticity approaches that for a loosely-bound crystal, suggesting that the high relaxation frequency is associated with motion inside a single potential well. (For the theory of the method, see following review).

R. M. Davies, Wales

**1396. H. T. O'Neil, "Reflection and refraction of plane shear waves in viscoelastic media,"** *Phys. Rev.*, Mar. 15, 1949, vol. 75, pp. 928-935.

This paper is concerned with the theory of the method recently developed by W. P. Mason et al, for the measurement of shear elasticity and viscosity of liquids in the high megacycle region (see preceding review). The method is based on the measurement of the attenuation and the phase change which a shear-stress wave in a solid suffers when it is reflected and refracted at a plane interface between the solid and the liquid under test.

To deduce the relationships required to interpret the experimental results, the author first discusses the propagation of sinusoidal waves in a viscoelastic medium in which the relations between stresses and strains and their time derivatives are linear. This is followed first by a general account of the reflection and refraction of plane shear waves with this type of medium, and then an analysis of the correlation of the properties of the medium with the quantities measured experimentally.

In the experiments, increased sensitivity was obtained by using incident waves directed obliquely at the interface and the theoretical treatment includes the effect of obliquity of incidence. An appendix gives a discussion of a restriction of the accuracy of the theory, which is shown to be unimportant under the actual experimental conditions.

R. M. Davies, Wales

## Mechanical Properties of Specific Materials

(See also Revs. 1390, 1408, 1458)

**1397. Karl Mathieu, "On the tensile strain behavior of austenitic steels at low temperatures"** (in German), *Arch. Eisenhüttenw.*, 1948, vol. 19, pp. 169-173.

A study was made to determine the phase changes of austenitic steels during tensile tests, and, in particular, a connection between the yield strength and the ( $\gamma$ - $\alpha$ )-transformation was established. The test materials were used in the form of small wires (0.0787 in. diam) of a Cr-Ni steel (0.18 Cr, 0.8 Ni), a Cr-Mn steel (0.176 Mn, 0.122 Cr) and a Mn steel (0.19 Mn, 0.12 Cr, 0.3 C). These steels change their phase condition only by simultaneous action of low temperature and cold-working.

The phase change occurring in a tensile test at +20 and -183 C was measured by magnetization with a field strength of about 180

amp-turns per cm and use of a ballistic galvanometer. From the stress-strain diagram and the corresponding magnetization curves for the three steels tested, it can be noted that a ( $\gamma$ - $\alpha$ )-phase change is closely connected with the appearance of a yield point. The less distinct the latter and the smaller the yield range of the steels, the less noticeable is the onset of the ( $\gamma$ - $\alpha$ )-transformation.

The author points out that considerable yielding is impossible without a phase change. The specific mechanical phenomena occurring during yielding (under tension and at low temperatures) are credited by the author to increased deformability of the material at the instant of the phase change.

M. Hempel, Germany

**1398. Harold C. R. Carlson, "Copper-base alloys for springs—parts 1 and 2,"** *Prod. Engng.*, 1949, vol. 20: Feb., pp. 103-107; Mar., pp. 87-91.

Part I presents general and specific data on the copper-base group of alloys used for spring materials. Discussions and data tables are given concerning corrosion, stress-corrosion cracking, gages, temper hardness, workability, and methods of joining of the alloys spring-brass, phosphor-bronze, and beryllium copper. Part II presents data (chemical composition, physical constants, and mechanical properties) for the same alloys, with discussions of heat-treatment, and applications of each alloy. Brief discussions are also given of five other copper-base alloys.

P. S. Symonds, USA

**1399. J. E. Burke and C. S. Barrett, "The nature of strain markings in alpha brass,"** *Trans. Amer. Inst. min. metall. Engrs.*, 1948, vol. 175, pp. 106-125.

Polycrystalline specimens of 70-30 alpha brass and single crystals of 80-20 alpha brass were subjected to plastic strains (mostly tensile) after which parts of the specimens were mechanically and electrolytically polished. The specimens were then etched to bring out the strain markings resulting from the deformations. It was found that the strain marks were equivalent, or closely related, to the usual slip bands caused by strain, and that strain markings could be detected as soon as slip lines were detected on the surface.

Much of the paper is concerned with whether the deformations were a result of mechanical twinning or slip. X-ray studies and other observations gave no evidence of twinning. It is concluded, from the direction of displacement as determined by the surface markings and from the magnitude of translation along a strain marking, that the deformation could be caused only by slip.

Irwin Vigness, USA

**1400. R. Maddin, C. H. Mathewson and W. R. Hibbard, Jr., "Unpredicted cross-slip in single crystals of alpha brass,"** *Trans. Amer. Inst. min. metall. Engrs.*, 1948, vol. 175, pp. 86-105.

Half-inch rods of 70-30 alpha brass were made into single crystals by the Bridgman method. These were polished mechanically and electrolytically and subjected to tensile loading of 100 lb per min. Load-extension measurements were made at frequent intervals. No work hardening was observed. Laue patterns were made after various amounts of plastic strain but no asterism was observed. The experimental observations were mainly of the appearance and characteristics of the slip lines, with especial attention to "cross-slip" lines that were frequently observed to connect the ends of nearby glide lamellae. It was observed that slip began at a resolved shear stress of about 1640 psi, after which clusters of slip lines appeared. Further deformation produced a

greater concentration of lines. It is suggested that the tendency of the lattice to rotate and the constraints imposed by the grips and unslipped matrix material sets up strains which are relieved by the cross-slipping. At very high magnification slip lines were observed about 340 atomic diam apart. Irwin Vigness, USA

1401. W. R. Hibbard, Jr., R. W. Fenn, Jr., Harold Margolin, and H. P. Moore, "Deformation lines in cold-rolled copper and its binary alpha solid solution alloys with aluminum, nickel and zinc," *Trans. Amer. Inst. min. metall. Engrs.*, 1948, vol. 175, pp. 74-85.

The paper reports observations of the effect of solute atoms on the minimum rolling reduction that must be given to a copper alloy before appreciable deformation lines (markings produced in the grains of a cold-worked metal by etching) are revealed. The minimum is greatly reduced by nickel and zinc, and particularly by aluminum. No correlation is found with grain size or with solution or strain hardening. It is suggested that the rate of etching may depend directly on the percentage of solute atoms and not necessarily only on disordering of the lattice by cold-work. Rodney Hill, England

1402. P. C. Varley, "The recovery and recrystallization of rolled aluminum of commercial purity," *J. Inst. Metals*, Dec. 1948, vol. 75, pp. 185-202.

When a cold-worked metal is annealed there is considerable reduction in strength before any observable recrystallization occurs. The authors refer to this as "recovery." By controlling the composition, casting methods, final cold-work, and previous hot-working and heat-treatment of polycrystalline aluminum sheet, the authors show that there is no discontinuity in the isothermal annealing curves which would indicate an onset of recrystallization. They find that recrystallization starts when the recovery has progressed to a point where the ultimate strength has reached some definite value. For the aluminum used this was  $7\frac{1}{2}$  tons per square inch. This value seems to be independent of the previous history of the sample.

The activation energy of the process was 51,240 cal per g atom. This also appears to be independent of the process variables and is the same whether measured in the recovery stage or after the onset of recrystallization. E. A. Davis, USA

1403. N. Dudzinski, J. R. Murray, B. W. Mott, B. Chalmers, S. F. Grover and W. Munro, "The Young's modulus of some aluminum alloys," *J. Inst. Metals*, Jan. 1948, vol. 74, pp. 291-314.

The desirability of finding aluminum alloys with a higher Young's modulus than those now available led to this investigation. The alloys studied were chosen only with the purpose of increasing the Young's modulus, no combination of desirable properties being sought.

Most of the experiments were conducted with the metal in chill-cast bar form. Among metals with one additional element besides the aluminum (binary alloys), manganese was most effective in increasing the modulus, followed in order by beryllium, cobalt, nickel, and silicon. Also investigated were ternary alloys, using largely the elements mentioned, and some quaternary alloys. One alloy was prepared in various conditions of fabrication, and others in the forged condition only but with alloying elements varied. Heat-treatment was applied to some. Preparation of the material and the effect upon Young's modulus are given in detail.

Testing was done by use of Lamb's extensometer on tensile test pieces. The alloys in the chill-cast condition were pre-

strained to not more than 0.1 per cent of the gage length. Sections from the various alloys were microscopically examined for their existing constituents.

The authors strongly urge that more work be done in this field, especially on other alloys. In an appendix, measurements of tension and compression moduli of many of the above alloys are presented in detail, and indicate that for aluminum alloys the moduli are equal. A. M. Zamboky, USA

1404. Julius Miklowitz, "The initiation and propagation of the plastic zone along a tension specimen of nylon," *J. Colloid Sci.*, Feb. 1947, vol. 2, pp. 193-215.

This paper is a report covering an experimental investigation of the behavior of nylon in tension. The effect of speed of loading, rigidity of the testing machine and the nature of the region of plastic yielding in the specimen are discussed.

Paul F. Chenea, USA

1405. Julius Miklowitz, "The stress-strain relationship of nylon under biaxial stress conditions," *J. Colloid Sci.*, Feb. 1947, vol. 2, pp. 217-222.

This paper is a continuation of previous studies by the author (see preceding review) in which the stress-strain relation for nylon under biaxial stress is investigated and compared with the results of the simple tension test.

Paul F. Chenea, USA

1406. A. G. H. Dietz, W. J. Gailus, and S. Yurenka, "Effect of speed of test upon strength properties of plastics," *Proc. Amer. Soc. Test. Mat.*, 1948, vol. 48, pp. 1160-1190.

Following an extensive historical review, a description of a new stress-strain machine is presented, and experimental results are given for cellulose acetate-butyrate, paper and laminated fabrics, and for polyvinyl chloride and polymethyl methacrylate.

George Halsey, USA

1407. L. J. Markwardt and J. A. Liska, "Speed of testing of wood: Factors in its control and its effect on strength," *Proc. Amer. Soc. Test. Mat.*, 1948, vol. 48, pp. 1139-1159.

Data are presented on the effect of rate of loading on the compressive strength and on the flexural strength of wood. The operation of various testing machines is considered at length.

George Halsey, USA

## Mechanics of Forming and Cutting

(See also Rev. 1401)

1408. Hugh Ford, "Researches into the deformation of metals by cold rolling," *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 39, pp. 115-143.

The paper describes an extensive series of measurements of roll force and roll torque in the cold rolling of copper and 0.2 per cent carbon-steel strip (without front and back tension), carried out on a 10-in diam experimental mill at Sheffield University by a team of the British Iron and Steel Research Association, headed by H. Ford.

The yield stress as a function of the plastic strain was obtained from tension and compression tests; it was used for comparing the measurements with theoretical calculations made by the methods of Ekelund, Orowan, and a new method developed by Bland and Ford. Owing largely to the low coefficient of friction (good lubrication), all methods give roll force values in good

agreement with the measurements in the practically important ranges of reductions; for the highest reductions, the Ekelund method gives too low values (because, being a first approximation, it does not taken into account the cumulative frictional effect). In the range of the variables investigated, the method of Bland and Ford gives practically the same values for the roll force as the Orowan method of which it is an approximate simplification. The calculated torque values are in less satisfactory agreement with measurements.

For the lowest coefficients of friction (smoothest roll surfaces) there is an approximate functional relationship between roll force and energy consumption for different reductions.

E. Orowan, England

**1409. D. R. Bland and Hugh Ford, "The calculation of roll force and torque in cold strip rolling with tensions,"** *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 39, pp. 144-163.

The Orowan method of calculating the distribution of roll pressure over the arc of contact does not make use of avoidable approximations, and therefore involves graphical or numerical integrations. The authors show how, by using a number of approximate assumptions which are satisfactorily fulfilled in typical cases of cold strip rolling, the Orowan method can be simplified to the extent that graphical or numerical integrations are avoided; for the ranges of the variables considered, the accuracy of the calculations without front and back tension is within about 15 per cent. The main basis of the method is the assumption

$$\left(\frac{s}{k} - 1\right) \frac{d}{d\phi} (hk) \ll hk \frac{d}{d\phi} \frac{s}{k},$$

where  $s$  is the normal roll pressure,  $k$  the yield stress,  $h$  the thickness of the rolled strip, and  $\phi$  the angular coordinate along the arc of contact.

E. Orowan, England

## Hydraulics; Cavitation; Transport

**1410. W. R. Purcell, "Capillary pressures—their measurement using mercury and the calculation of permeability therefrom,"** *J. petrol. Tech.*, Feb. 1949, vol. 1, pp. 39-48.

The paper describes a method and procedure for determining capillary pressure as a function of liquid content for porous media. The pressure-volume curves resulting from forcing mercury into a highly evacuated porous-medium specimen of any shape are shown to be similar to capillary-pressure curves obtained by the conventional porous-diaphragm technique. The method has the advantage that capillary-pressure curves may be obtained in a matter of hours as compared to several weeks required with the diaphragm method.

Data for a wide variety of porous media are presented for comparison of the mercury method and the conventional air-water displacement technique. The experimental results indicate that reasonably close agreement can be achieved.

The paper also presents a calculation procedure wherein capillary-pressure curves may be used to determine the homogeneous-fluid permeability of porous media. Data are presented for comparison between calculated permeabilities and experimentally determined values. The equations provide a fairly reliable method of calculating permeability.

A combination of the method for measuring the capillary-pressure curve of drill cuttings and the equation relating permeability to the capillary-pressure so determined makes possible the estimation of the permeability of these cuttings.

Although the mercury-imbibition technique has been used for some time for determining pore-size distributions of catalysts, the applications discussed in this paper are important to the field of petroleum technology. The method cannot, in its present state of development, supplant completely the techniques which employ brine, oil and gas because the interactions between these fluids and the clay constituents of the porous media differ from those obtained with mercury.

John A. Putnam, USA

**1411. R. A. Collacott, "Discharge coefficients of chamfered orifices and nozzles,"** *Aircr. Engng.*, Apr. 1948, vol. 20, pp. 112-113.

The paper gives the results of an experimental investigation of the effect of angle and depth of inlet chamfer on the variation of the discharge coefficient  $C_D$  of a nozzle of  $\frac{1}{2}$  in. bore and  $2\frac{1}{8}$  in. length and an orifice of  $\frac{1}{2}$  in. bore and  $\frac{1}{8}$  in. thick. The head applied varied between 4 in. and 20 in. Experiments were made with: (a) constant face diameter of chamfer ( $\frac{9}{16}$  in.) and variation of angle of chamfer; (b) constant angle of chamfer (90 deg) and variation of bevel diameter from  $\frac{9}{16}$  in. to  $\frac{3}{4}$  in.

The results were as follows: (a)  $C_D$  for the nozzle increased with increasing bevel angle to a maximum value of 0.95 at an angle of 95 deg (20 in. head); the bevel angle for maximum  $C_D$  increased with decreasing head while the maximum  $C_D$  decreased. For the orifice, a maximum  $C_D$  of 0.87 for a 20 in. head was reached at a bevel angle of 55 deg; decrease in head increased the maximum  $C_D$  almost linearly to 0.907 at a head of 4 in. and an angle of 68 deg. (b) Varying the depth of a 90-deg chamfer had little effect on the discharge coefficient of either nozzle or orifice.

The results are discussed on the basis of the flow conditions existing in the orifice and nozzle. Graphs are given for anticipated variations of  $C_D$  with depth of chamfer, and for angles of 60, 90, and 120 deg, for both orifice and nozzle.

Hans F. Winterkorn, USA

**1412. John Allen and Wilfred Eastwood, "Scale-model experiments to determine the best form of silt-trapping basin at the confluence to the Manchester ship canal and the river Mersey,"** *J. Instn. civ. Engrs.*, Jan. 1949, vol. 31, pp. 241-269.

In recent years, the Manchester Ship Canal Company has spent considerable sums on the dredging of sand and silt carried over the Irlam weir by the River Mersey and deposited in the ship canal. Model studies were undertaken to help in the design of a settling basin of sufficient capacity. This basin was also to enable ships up to 450 feet in length to turn around in the basin. Six possible designs were subjected to tests and various modifications were tried.

The model investigation indicated that a very substantial portion of the silt could be trapped in a basin of comparatively moderate dimensions. The beneficial effect of a vertical baffle wall just downstream of the weir was demonstrated. The amount or rate of feed of granular material (powdered pumice) into the model did not have an appreciable bearing on the proportion deposited in the settling basin.

After completion of the first studies, it became necessary to explore the possibilities of another site for the stilling basin, the area previously used being required for a power station. The second series of tests indicated a scheme giving the highest deposits in the basin and the lowest deposits in the canal, but later modifications in the area available for the basin made it impossible to use that scheme and a second-best scheme had to be finally chosen.

André L. Jorissen, Belgium

## Incompressible Flow: Laminar; Viscous

(See also Revs. 1395, 1396, 1411, 1440)

1413. W. P. Jones, "Note on lifting plane theory with special reference to Falkner's approximate method and a proposed electrical device for measuring downwash distributions," *Rep. Memo. aero. Res. Council. Lond.*, no. 2225, May 1946 (issued in 1947), pp. 1-4.

A brief description is given of the theory [same source, no. 2145, May 1943] concerning the calculation of the downwash on a thin wing of arbitrary plan form and lift distribution in an incompressible, inviscid fluid as given by a distribution of doublets over the wing and its wake. A further brief description is given of the method of Falkner [same source, no. 1910, Aug. 1943], in which the solution is approximated by a number of chordwise strips, each with spanwise constant doublet strength and four chordwise staggered doublet layers of constant strength.

The two methods are compared on the basis of downwash calculations of three lift distributions on a rectangular wing of aspect ratio 6. Reasonable agreement is found at mid-chord up to about 0.8 span.

It is suggested that the magnetic analogy might be useful in determination of downwash by measuring the magnetic field obtained from a small finite number of horseshoe-shaped current-carrying conductors.

W. G. Cornell, USA

1414. C. L. Pekeris, "Stability of the laminar parabolic flow of a viscous fluid between parallel fixed walls," *Phys. Rev.*, July 15, 1948, vol. 74, pp. 191-199.

A stability analysis is made for the case of two-dimensional laminar flow between parallel plates affected by small two-dimensional disturbances. Asymptotic expressions are found, for the damping factor and the propagation velocity, for small  $\alpha$  ( $\alpha = 2\pi h/\lambda$ , where  $\lambda$  is the disturbance wave length and  $2h$  is the plate spacing) and large  $\alpha R$ , where  $R$  is the Reynolds number of the basic laminar flow. The asymptotic expressions for first-mode perturbations were checked by exact methods for  $\alpha R = 5430$ , showing excellent agreement. Stability of the first mode is found, in disagreement with previous results of Lin [*Quart. appl. Mech.*, 1946, vol. 3, p. 288] and Meksyn [*Proc. roy. Soc. Lond. Ser. A*, 1946, vol. 186, p. 391].

Two classes of perturbations are found: the first where propagation velocity approaches zero for  $\alpha^2 = 0$  and  $\alpha R \rightarrow \infty$ , the second where propagation velocity approaches the maximum velocity (channel center) of the basic laminar flow. All modes of the second class are found to be stable for large  $\alpha R$ . All modes of the first class are stable if the first two terms in the asymptotic expression for damping factor give a valid representation.

In the case of even modes of the first class, negative propagation velocity (upstream propagation) is found for  $\alpha^2 R > M$ , where  $M = 10,700$  for the second mode. The effect of this result on stability experiments is discussed. William G. Cornell, USA

## Compressible Flow, Gas Dynamics

(See also Revs. 1434, 1435, 1436, 1442, 1459)

1415. P. A. Lagerstrom, J. D. Cole and Leon Trilling, "Problems in the theory of viscous compressible fluids," California Institute of Technology, Pasadena, 1949. Paper, 8.2 x 10.8 in., 200 pp.

This reports part of a wider program for exploring viscous compressible flow patterns. Bold extensions of the usual concept of boundary-layer or shock-wave theory are needed to deal with

some of these phenomena. This book deals with much simpler questions, and does not supply experimentally verifiable predictions. However, considerable insight is given into the physical mechanism of viscosity and its special mathematical properties.

First, the characteristics of the complete system of equations for one-dimensional unsteady flow are determined for the most general case, and also for the cases of zero viscosity, zero heat conductivity or both zero. Some results are surprising: Even with heat conductivity and friction, one of the family of characteristics is given by the streamlines. The fact that for zero friction certain families of characteristics existing with friction are discontinuously replaced by others, leads to the question of what significance the characteristics for zero viscosity have in the case of small viscosity. In a later section (Singular Perturbation Problems) an example is given which shows that for small viscosity the essential changes of state propagate in the vicinity of a characteristic for zero friction.

Next, the equations of motion are linearized by assuming that velocities and other changes of state are small and the heat conductivity is zero. (The case of small deviations from a parallel flow is connected to this by transformation of coordinates). For the linearized equation, each solution can be divided uniquely into two portions satisfying the equations separately: for the first portion (called the longitudinal wave) the velocity field is irrotational; for the second portion (transversal wave) the velocity field is without divergence. The simplest examples are one-dimensional unsteady transversal or longitudinal waves (unsteady motion of an infinite plate parallel or normal to its surface).

The results are used to illustrate the previous singular perturbation problems. Hints for extending these to nonlinear cases are given. As the simplest examples for higher dimensional solutions the flows produced by an expanding sphere or cylinder (longitudinal waves) or by a rotating cylinder (transversal waves) are considered. From these, other expressions are derived by differentiation with respect to a space coordinate, which can be superimposed and combined with a Galilean transformation to form a steady flow with finite velocity at infinity. One sees that the conditions along a flat plate (even if infinitely short) cannot be fulfilled by transversal waves alone, the coupling between the two types being given by the boundary conditions. In the terminology of boundary-layer theory the transversal wave roughly corresponds to boundary-layer flow, and the longitudinal wave to the flow changes caused by the displacement thickness of the boundary. The connection between the boundary-layer solutions and transversal waves is discussed.

The solution for an infinitely short, flat plate with a finite tangential force is first approximated by an iteration procedure, and the exact expression is then found by using a rather general theorem of Plejel. The connection between these approaches is discussed in detail. Finally, the problem of the flat plate is formulated and different ways of solving it are mentioned. The appendixes give the mathematical background for these discussions and some interesting extensions to nonlinear cases.

Gottfried Guderley, USA

1416. R. E. Meyer, "The method of characteristics for problems of compressible flow involving two independent variables—Part II," *Quart. J. Mech. appl. Math.*, Dec. 1948, vol. 1, pp. 451-469.

In this second of two papers the author applies the theory developed in part I [see Rev. 1516, Oct. 1948] to the case of axisymmetric flow. First- and second-order differential equations are derived for the propagation of discontinuities in the space derivatives along the Mach lines.

Specializing in the case of a uniform flow entering a contractor, the author uses the above differential equations to find the first and second terms of the Taylor expansion for the velocity field induced by the initial curvature and form of the contractor. This method of approximation is compared with the use of linearized field equations. Conditions for shock-free flow at the entrance of a contractor are also given. H. G. Elrod, Jr., USA

**1417. J. Kestin and A. K. Oppenheim, "The calculation of compressible fluid flow by the use of a generalized entropy chart,"** *Proc. Inst. mech. Engrs.*, 1948, vol. 159, no. 43, pp. 313-334.

This paper provides a comprehensive graphical means of solving problems involving steady one-dimensional adiabatic flows of gases having constant specific heats. The "generalized entropy chart" is an extension of the classical Mollier diagram to permit the reading off of all necessary dynamic flow quantities (such as Mach number, mass flux density, momentum per sec, etc.) in addition to the usual purely thermodynamic quantities. The method is not confined to isentropic flows but may be applied to problems involving friction losses and shocks. Because of its graphical nature the method is most valuable when qualitative or only moderately accurate quantitative results are required.

R. C. Prim, USA

**1418. A. Robinson, "On source and vortex distributions in the linearized theory of steady supersonic flow,"** *Coll. Aero. Cranfield Rep.*, no. 9, Oct. 1947, 28 pp.

In this highly mathematical paper the author discusses the singularities in the solutions of the linearized supersonic-flow equations by the use of Hadamard's concept of the "finite part of an infinite integral." After a review of this concept, the rules for the calculation of finite parts and the extensions of the theorems of Gauss, Green and Stokes to finite parts are given. Line, surface and volume distributions of sources and doublets, and various line, surface and volume integrals associated with these distributions are considered. A discussion of vortex distributions concludes the paper.

The author states that applications of these considerations to airfoil theory are to be given in a separate report (see following review).

Paul A. Libby, USA

**1419. A. Robinson and J. H. Hunter-Tod, "Bound and trailing vortices in the linearised theory of supersonic flow, and the downwash in the wake of a delta wing,"** *Coll. Aero. Cranfield Rep.*, no. 10, Oct. 1947, 17 pp.

The present report is a continuation and an application of a previous paper by Robinson (see preceding review). These papers give a careful formulation of supersonic airfoil theory along lines that are intended to preserve, in so far as possible, the older, familiar concepts of subsonic, or incompressible, wing theory. The mathematical tools required in this formulation of the supersonic wing theory are of comparatively recent origin (Hadamard, M. Riesz). It seems that in order to retain the familiar concepts of "source," "doublet," and "vortex" in supersonic studies the aerodynamicist must acquaint himself with newer definitions in the application of divergent integrals. [For a parallel development of the theory, as well as its application to this particular downwash problem, the reader may consult papers by Heaslet and Lomax, *Nat. adv. Comm. Aero. tech. Notes*, nos. 1515 and 1620; see Rev. 500, Mar. 1948]. In addition to the "finite part" of a divergent integral as introduced by Hadamard, the present authors further propose definitions of "hyperbolic curl" ( $\text{curl } \Phi$ ) and "hyperbolic divergence" ( $\text{div } \Phi$ ).

In addition to the theoretical treatment, the paper presents graphs of the downwash along the center line of the wake for a variety of triangular wings at supersonic speeds.

Robert T. Jones, USA

**1420. Max A. Heaslet, Harvard Lomax, and John R. Spreiter, "Linearized compressible-flow theory for sonic flight speeds,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1824, Mar. 1949, pp. 1-45.

It had previously been noted (for example, in the theory of the lift of a delta wing, the drag of delta wings with swept trailing edges and the lift due to a sudden change in angle of attack of a two-dimensional wing) that the solutions of many supersonic wing problems by linearized methods show finite (small) disturbances even in the limit  $M = 1$ , where  $M$  is the free-stream Mach number. The authors of this paper suggest that, since these limits show small disturbances, the linear theory is still valid for this type of problem at and near  $M = 1$ . The authors then discuss in detail several problems of this type.

Special problems that are discussed are the response of a two-dimensional wing to a sudden change in angle of attack (produced without rotation of the wing). By means of the analogy with a supersonic three-dimensional steady flow, the results for slightly subsonic, sonic and supersonic motion are treated. The sonic flow solution is generalized by superposition methods to discuss the lift for a "square-wave" angle of attack variation with time and the lift for a sinusoidal oscillation starting suddenly at an initial instant.

A generalized treatment is given of some three-dimensional steady-state wing problems by consideration of the behavior in the region at infinity of the corresponding source and doublet fields. The drag of an infinite swept wing with a double-wedge profile at  $M = 1$  is discussed. The lift of a delta wing and of a swept wing with straight leading edges at  $M = 1$  is also determined.

H. J. Stewart, USA

**1421. Stefan Bergman and Bernard Epstein, "Determination of a compressible fluid flow past an oval-shaped obstacle,"** *J. Math. Phys.*, Jan 1948, vol. 26, pp. 195-222.

The problem of finding the flow past a given body is first discussed in general terms. In particular, it is pointed out that such a problem is very difficult even in the incompressible case. The authors then turn to the hodograph method, in the manner developed by Bergman.

Certain tables which are essential in that method are prepared. With the help of these tables, it is shown how the flow past an oval shaped obstacle can be determined. One example is worked out to the extent of determining approximately the form of the obstacle and one of the stream lines.

C. C. Lin, USA

**1422. A. D. Young, "Note on the velocity and temperature distributions attained with suction on a flat plate of infinite extent in compressible flow,"** *Coll. Aero. Cranfield Rep.*, no. 8, 1947, 9 pp.

Formulas are derived by means of which the velocity and temperature distributions for compressible flow for the title case may be calculated for any given Mach number, it being assumed that there is no heat transfer by conduction at the plate. The method consists of establishing a correspondence between the velocity and temperature profiles for incompressible flow and those for compressible flow, the lateral ordinates being scaled by factors which are functions of the ordinates and of the Mach number.

Curves of the velocity and temperature distributions covering a range of Mach numbers up to 5.0 are included, for both a linear and a 0.76 power variation of viscosity with absolute temperature.

Nicholas Di Pinto, USA

1423. J. Fabri, "On an approximate method of calculating the load distribution on airfoils with supersonic leading edges" (in French), *C. R. Acad. Sci. Paris*, Apr. 12, 1948, vol. 226, pp. 1172-1174.

A method is presented for the calculation of the pressure distribution on a wing in supersonic flow. It yields an approximation to the result obtained by linearized theory, and is based on the assumption that the variation of the local incidence at any given point of the airfoil induces inside its Mach cone the same pressures as under two-dimensional conditions, i.e., as by Ackeret's theory. Examples are given which show that good agreement with exact linearized theory is obtained for steady flow around wings with supersonic leading edges.

A. Robinson, England

1424. Frank S. Malvestuto, Jr., Kenneth Margolis, and Herbert S. Ribner, "Theoretical lift and damping in roll of thin sweptback wings of arbitrary taper and sweep at supersonic speeds—subsonic leading edges and supersonic trailing edges," *Nat. adv. Comm. Aero. tech. Note*, no. 1860, Apr. 1949, 40 pp.

Using linearized theory, the lift-curve slope and roll-damping derivatives for tapered, swept wings having subsonic leading edges and supersonic trailing edges are calculated. Results are presented in the form of design charts with aspect ratio, taper ratio, leading-edge sweep, and Mach number as parameters. The pressure distribution within the wing-tip Mach cones is calculated approximately by Evaard's method, while the pressure distribution elsewhere on the wing follows from the previously known results for a triangular wing.

Despite the authors' statement to the contrary, the results for the roll-damping derivative are valid for a wing with arbitrary twist and camber, in so far as the linearized theory is valid.

John W. Miles, USA

1425. A. Robinson, "On some problems of unsteady supersonic aerofoil theory," *Coll. Aero. Cranfield Rep.*, no. 16, May 1948, 22 pp.

Unsteady supersonic flow round an airfoil of infinite span is considered in the first part of the paper. Linearized theory is used throughout. It is shown that the pressure at any point of the surface under forward acceleration can be analyzed into three components, one of which is the steady (Ackeret) pressure due to the instantaneous velocity, while of the other two, one depends directly on the acceleration, and one on the square of the velocity during a limited time interval preceding the instant under consideration. However, the difference between the total pressure and the "steady-pressure component" is such that it can be neglected in all the definitely supersonic conditions ( $M \geq 1.15$ , say) that are likely to occur in practice. The form in which the result is presented differs from that in a recent paper by Gardner and Ludloff (presented at the annual meeting, Institute of the Aeronautical Sciences, January 1949, available as preprint no. 186), but the magnitude appears consistent.

The oscillatory supersonic flow round a delta wing inside the Mach cone emanating from its apex is considered in the second part of the paper. Particular "normal" solutions are obtained by means of a special system of pseudo-orthogonal coordinates. Each such solution is of the form of a product of a Bessel func-

tion, a Lamé function of the first kind, a Lamé function of the second kind, and an oscillatory factor. It is shown that the velocity potentials corresponding to vertical and pitching oscillations of the wing can be represented by an infinite series of such normal solutions. The results are apparently not limited to small oscillation frequencies, but their practical use will entail considerable labor.

H. S. Ribner, USA

1426. G. Guderley, "On the transition from a transonic potential flow to a flow with shocks," *Hdqtrs. Air Mat. Comm. Dayton Tech. Rep.*, no. F-TR-2160-ND, Aug. 1947, 46 pp.

The author deals with the problem of the disturbances that arise if the boundary of a known transonic flow pattern, i.e. mixed subsonic and supersonic flow, is slightly deformed. Two types of disturbances can occur. The first has essentially the behavior of disturbances in subsonic flow; it vanishes at a distance from the surface. The second, having the behavior of a supersonic disturbance, is propagated downstream and may tend to infinity toward the end of the supersonic region. This second type produces shock as the originally smooth flow tends to become extremely wavy as the end of the supersonic region is approached.

This general picture of mixed flow is obtained from a rigorous mathematical treatment of the basic flow equations, using the hodograph equations. After setting up the equations and discussing the transonic law of similarity, the boundary conditions are examined. The problem is restricted to the flow in the neighborhood of a known field of flow to avoid the difficulty of the boundaries in the hodograph plane. As an example, the flow near a potential vortex is chosen with two coaxial circles as flow boundaries, the inner one being deformed and the outer one allowed to tend to infinity. This example is generalized and the problem of disturbances in a general mixed flow examined. A section dealing with the effect of a subsonic boundary layer leads to the conclusion that it has no influence on the magnitude of the disturbances.

The paper admittedly deals only with the first occurrence of shock, but this is shown to be inevitable, a conclusion of some importance.

R. C. Knight, England

1427. G. Guderley, "Nonstationary gas flow in thin pipes of variable cross section," *Nat. adv. Comm. Aero. tech. Memo.*, no. 1196, Dec. 1948, 81 pp.

This translation of a German 1942 ZWB report (FB 1744) deals with stepwise computation of the one-dimensional case of the title flow, the velocity  $v$  being assumed constant over any cross section of the pipe. Details are worked out for perfect gases only. The characteristic equations (called here "compatibility conditions") are shown to be

$$dy = (v \pm a)dt, \quad dp \pm \rho v dv = -a^2 p(F_y v + F_t)dt/F \quad (*)$$

where  $y$  is the particle abscissa,  $a$  the speed of sound, and  $F$  the cross-sectional area.

The general case is prepared by a discussion of isentropic flow in ducts with constant, then with variable cross section. In the former case the (now familiar) quantities

$$\lambda, \mu = v \pm \int di/a$$

( $i$  is the enthalpy) are introduced and shown to be the characteristic parameters. They are kept as the dependent variables (but no longer as characteristic parameters) for the general case of nonisentropic flow of an ideal gas and a duct of variable cross section (in this case  $i$  and  $a$  are still functions of the temperature alone). The third equation expresses constant entropy of a

particle. The now familiar triangulation is then explained.

This reviewer does not understand the devotion to  $\lambda$  and  $\mu$  in the general case: (1) they are not necessary for the triangulation procedure; (2) they lead to characteristic equations more involved than (\*); and (3) they add to the number of quantities that must be computed.

More than one half of the detailed discussion is devoted to shock computation (in the well-known terms introduced by the author:  $\pi = \exp(s_0/R - s/R)$  and  $\psi$ , the mass abscissa of the particle). Shock conditions are rewritten in terms of  $\lambda$ ,  $\mu$ ,  $\pi$  and  $\psi$ . Only "attached" shocks (e.g., velocity jump of a piston) are discussed. A stepwise procedure is described, then a certain iteration procedure.

The reviewer's reasonable effort to understand this part of the translation was unsuccessful, in spite of the practice offered by the preceding pages. The translation is quite representative of the average present-day product, and its quality is a corollary of the regrettable fact that it is evidently impossible to coax a competent person even into editing a translation.

A. W. Wundheiler, USA

**1428. M. Schaefer, "Outflow of a jet of compressed air into moving air,"** *Hdqtrs. Air Mat. Comm. Dayton Transl.*, no. A9-T-15, Jan. 1949, 14 pp.

The paper indicates a procedure for calculating the jet of compressed air issuing forth from a nozzle of prescribed dimensions at the stern of the German A-4 body under the influence of the external flow. To simplify the calculations it is assumed (1) that an exact conical field of flow exists within the nozzle, (2) that the exit cross section of the nozzle coincides with the end cross section of the stern thus avoiding a dead-water region, (3) that the stern of the body can be replaced by a truncated cone about which a parallel flow takes place at constant Mach number  $M = 3.24$ , and (4) that the flow after the compression shock is an adiabatic potential flow.

None of the equations used for the calculation is presented in the report. Basically it is indicated that the solution of the flow field is calculated by the lattice-point method of characteristics and compression-shock theory. Since the diagram of the flow construction and the table of calculations are missing from the report, its value is lessened considerably.

Lester L. Cronvich, USA

**1429. Numa Manson, "On the theory of the propagation of deflagrations"** (in French), *C. R. Acad. Sci. Paris*, Oct. 11, 1948, vol. 227, pp. 720-722.

Earlier contributions [same source, 1948, vol. 226, no. 69, p. 163] have discussed the effect of flow perturbations accompanying ignition on the propagation of deflagrations in gas mixtures contained in circular tubes. In the present note, these results are extended. It is shown that the perturbation accompanying ignition is propagated as a pressure pulse with overpressure of the same order of magnitude as the pressure drop across the deflagration wave, and a velocity greater than the deflagration velocity. Owing to reflection on the deflagration front, there results a pair of such pulses in advance of the deflagration wave. The second pulse can overtake and reinforce the first, giving rise to a single steep-fronted pressure pulse. The details of the phenomenon depend upon the nature of the gas and differ depending on whether the cylindrical container is open or closed. These results are in accord with the hypothesis that the transition from deflagration to detonation is effected by the formation of a shock wave in advance of the deflagration wave.

Stuart R. Brinkley, Jr., USA

**1430. A. M. Fainzilber, "Questions of thermal modelling"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Feb. 1, 1948, vol. 59, pp. 697-700.

The present paper deals with the problem of the thermal phenomena occurring in an aggregate of viscous fluids in motion. Starting with the equation of the conservation of energy and the hydrodynamic equation, there results the differential equation

$$\tau T_{uu} + (1 - P_r)\tau_u T_u + (\mu - 1)P_r B^2 \tau = 0 \quad (*)$$

in which  $u$ ,  $T$  are the ratios of velocity and temperature to their values in adiabatic flow, respectively;  $\tau\mu^{1/2}$  is the intensity of viscous shear;  $\mu$  is the kinematic viscosity;  $\rho$  is the density;  $P_r$  is Prandtl's number; and  $B$  the Mach (Barstow) number. Putting  $\tau = A_0(1 - u^3)$  so as to satisfy the boundary conditions, Equation (\*) reduces to:

$$T_{uu} - 3bu^3 T_u / (1 - u^3) = 0$$

[ $b = 1 - P_r$ ;  $a = (\mu - 1)P_r B^2$ ], a solution of which is given in the form of a series for  $T$  in powers of  $u$ . Tables of  $T$  are set up for water vapor and air and conclusions drawn concerning the temperature distribution obtaining in a mixture of air and water vapor in motion.

In the case  $B < \frac{1}{2}$ , the motion of the fluids can be taken as being isothermal. Introducing the total energy as the dependent variable  $Q = \frac{1}{2}u^2 + \int \rho^{-1} dp$ , and  $X = \mu \int \rho dx$ ,  $Y = \psi(x, y)$  as the independent variables [ $\psi(x, y)$  being the stream function of Stokes] the general equation becomes

$$\partial Q / \partial X = \{2(Q - Q_0)\}^{1/2} \partial^2 Q / \partial Y^2. \quad (**)$$

The energy  $Q$  is measured from its value corresponding to an adiabatic flow taken as zero. The boundary conditions applicable to (\*\*) are  $Q = Q_0$  at  $Y = 0$  and  $Q = 0$  at  $Y = \infty$ . Equation (\*\*) is general in the sense that it holds for any fluid, the various viscous continuums differing only in the form of the function  $X$ .

Michael Daniloff, USA

**1431. Robert T. Jones, "Wing plan forms for high-speed flight,"** *Nat. adv. Comm. Aero. Rep.*, no. 863, 1947 (issued in 1949), 5 pp.

This report shows that for airfoils of infinite aspect ratio moving at an angle of sideslip, the pressure distribution is determined only by the component of motion normal to the leading edge. The attachment of plane sound waves may be eliminated and the resulting pressure drag may be reduced at near-sonic and supersonic speeds by plan forms with sweep angles greater than the Mach angle. For aerodynamic efficiency at supersonic speeds the sweepback should be greater than the Mach angle and should be such that the component of velocity normal to the leading edge is less than the critical speed of the airfoil sections. This principle should also include considerations of the induced velocities resulting from the thickness of the airfoil, particularly at high subsonic speeds.

H. P. Lapman, USA

## Turbulence, Boundary Layer, etc.

(See also Revs. 1414, 1415, 1422, 1426, 1438)

**1432. R. C. L. Bosworth, "Distribution of reaction times for turbulent flow in cylindrical reactors,"** *Phil. Mag.*, Mar. 1949, vol. 40, pp. 314-324.

The time which a molecule spends in a cylinder through which there is turbulent flow is given by a frequency curve which has a sharp peak at a value slightly greater than that for the central core. This curve has been here calculated on the assumption

that eddy diffusion may be taken by Reynolds' analogy as determined by the velocity gradient, using the Prandtl-von Kármán form for this function, and a power-law velocity distribution with an exponent varying with the Reynolds number. The ratio of the length of the cylinder to its diameter is the principal variable.

The reviewer questions the identification of the time which a molecule spends in a reactor with the time available for the reaction, except in unimolecular reactions. Benjamin Miller, USA

1433. G. K. Batchelor, "Energy decay and self-preserving correlation functions in isotropic turbulence," *Quart. appl. Math.*, July 1948, vol. 6, pp. 97-116.

This paper contains a critical review and some extensions of several existing assumptions of self-preservation of the correlation functions in isotropic turbulence and the law of decay thus obtained. Emphasis is placed on the fact that a complete similarity of the correlation function can only exist and will exist at low Reynolds numbers. It is pointed out that the longitudinal correlation function  $f(r, t)$  must then be of the form  $\exp(-r^2/8\nu t)$ , where  $\nu$  is the kinematic viscosity coefficient. This is shown to be the only solution of the von Kármán-Howarth family with a Loitziansky invariant neither zero nor infinity.

C. C. Lin, USA

1434. M. W. Rubesin and H. A. Johnson, "A critical review of skin-friction and heat-transfer solutions of the laminar boundary layer of a flat plate," *Trans. Amer. Soc. mech. Engrs.*, May 1949, vol. 71, pp. 383-388.

The available literature on the drag and heat-transfer coefficients of a flat plate having a uniform surface temperature and immersed in a uniform gas stream at zero angle of attack is briefly reviewed. It is stated that the effects of plate temperature and Mach number on the heat transfer and drag coefficients for plates in air are not given correctly when the viscosity law  $\mu_1/\mu_2 = T_1/T_2$ , where  $\mu$  is the viscosity and  $T$  is the absolute temperature, is used.

Curves of  $C_D R^{1/2}$  against Mach number, where  $C_D$  is the drag coefficient and  $R$  is the Reynolds number, calculated by various investigators for different values of the Prandtl number and for different values of the exponent  $n$  in the relation  $\mu_1/\mu_2 = (T_1/T_2)^n$ , are presented both for the insulated plate and for the plate with heat transfer. The results are briefly discussed. A relation between the heat-transfer coefficient and the drag coefficient for the plate with heat transfer is derived. The reviewer notes that it is believed that the given relation between the heat-transfer and drag coefficients implies a constant density and viscosity, an assumption inconsistent with the use of a variable density and viscosity in the calculation of drag coefficients.

Finally, relations are presented for the calculation of a reference temperature  $T_1$ , having the property that  $C_D R_1^{1/2}$  becomes approximately independent of the Mach number and the ratio of plate to stream temperature when the density and viscosity used in  $C_D$  and  $R_1$  are based on  $T_1$ . Neal Tetervin, USA

## Aerodynamics of Flight; Wind Forces

(See also Revs. 1374, 1413, 1419, 1420, 1424, 1431, 1443)

1435. A. Robinson and J. H. Hunter-Tod, "The aerodynamic derivatives with respect to sideslip for a delta wing with small dihedral at supersonic speeds," *Coll. Aero. Cranfield Rep.*, no. 12, Dec. 1947, 24 pp.

Expressions are derived for the sideslip derivatives on the assumptions of the linearized theory of flow for the title case. The

sideslip is sufficiently small that if the wing is initially wholly within the Mach cone emanating from its apex, it will remain so in the disturbed condition and vice versa.

When the leading edges are within the Mach cone from the apex, the pressure distribution and the rolling moment are independent of Mach number but dependent on aspect ratio. There is a leading-edge suction, which is a function of the incidence, aspect ratio and Mach number, which contributes as well as the surface pressure distribution to the side-force and yawing moment.

When the leading edges are outside the apex Mach cone, the nondimensional rolling derivative, is, in contrast to the other case, dependent on Mach number and independent of aspect ratio; the other derivatives and the pressure, however, depend on both variables. There is no leading-edge suction force in this case.

A transformation is derived that links the methods of H. J. Stewart and A. Robinson under conditions of conical flow. Stewart's method is shown to be suitable for calculating the aerodynamic derivatives with respect to sideslip, and Robinson's method for calculating the pitching moment due to pitching and the rolling moment due to rolling. H. Reese Ivey, USA

1436. Donald J. Graham, "The development of cambered airfoil sections having favorable lift characteristics at supercritical Mach numbers," *Nat. adv. Comm. Aero. tech. Note*, no. 1771, Dec. 1948, 82 pp.

The development of a new 8 series of NACA airfoil sections with improved lift characteristics near the design lift coefficient at supercritical values of Mach number is described. These airfoils, designated as the NACA 8 series, have approximately equal values of critical Mach number on their upper and lower surfaces at the design lift coefficient. Wind-tunnel tests show that the angle of attack required to maintain the design lift coefficient decreases up to a Mach number of 0.90 (the limit of the tests). However, the variation of slope of the lift curve with Mach number is not improved over that of the standard NACA 6 series of airfoils, and the drag characteristics of the new airfoils are less favorable than those of the 6 series airfoils. It is shown that the favorable supercritical lift characteristics of the proposed airfoils can be approximated by deflecting a trailing-edge flap upward by 6 deg on a standard 6 series airfoil.

Tables of ordinates and theoretical velocity distributions for 8 series airfoils having thickness ratios of 10 and 16 per cent are included. W. O. Breuhaus, USA

1437. A. Robinson and F. T. Davies, "The effect of the sweep-back of delta wings on the performance of an aircraft at supersonic speeds," *Coll. Aero. Cranfield Rep.*, no. 6, Mar. 1947, 17 pp.

The total drag of a hypothetical supersonic airplane is calculated with the angle of wing sweepback as a parameter, for  $M = 1.2$  and 2 and at altitudes from 10,000 to 80,000 ft. The existence of an optimum sweepback angle depending upon speed and altitude is illustrated. J. M. Wild, USA

1438. H. Guillemet and P. Guienne, "Method of calculating the minimum drag of airplanes" (in French), *Off. nat. Étud. Rech. aéro. tech. Note*, no. 43, 1948, 40 pp.

Two semiempirical formulas due to Weinig for computing the drags of struts and bodies of revolution, in which both the shape parameters of the body and the Reynolds number are taken into account, are discussed. The formulas, and modifications of them by others, are criticized by the authors for failure to consider possible variations in the transition point in the boundary layer.

They propose to retain Weinig's formulas but with the restriction that one of the factors in the formulas, the friction drag of an equivalent flat plate, be that for a flat plate having the same relative point of transition as the profile being considered.

L. Landweber, USA

**1439. Walter J. Klinar and Thomas L. Snyder, "Influence of tail length upon the spin-recovery characteristics of a trainer-type-airplane model,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1764, Dec. 1948, 17 pp.

Tests conducted in the NACA 20-ft free-spinning tunnel on a model having two different tail lengths indicated that spin recovery characteristics were improved by the longer tail length, even though the tail area was so modified that the tail-damping-power factor was greater with the shorter tail length. Mass and moment of inertia characteristics were maintained the same with both tail length configurations. It was concluded that spin recovery characteristics may be influenced by tail length to a greater extent than is indicated by the value of the tail-damping-power factor, and that increasing tail length is a more effective means of improving spin recovery than is the addition of ventral-fin area.

W. O. Breuhaus, USA

**1440. S. U. Bencotter, "Impact theory for seaplane landings,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1437, Oct. 1947, 52 pp.

The forces on a seaplane hull during its initial landing impact are determined on the basis of the usual von Kármán strip theory of the hydrodynamic (apparent mass) forces on the hull, including certain modifications, due to W. L. Mayo (*same source*, no. 1008), to account for the momentum imparted to the fluid in the wake when there is motion parallel to the keel. The strip apparent mass is calculated as that of a flat plate of width equal to the instantaneous width of the hull at the water line. Any forces due to buoyancy, viscosity, and surface waves are neglected. The modifications of the apparent mass obtained from strip theory, caused by effects of piled-up water, hull aspect ratio, and deviation of the strip apparent mass from that of a flat plate, are accounted for by a semiempirical formula given by Mayo.

The author succeeds in obtaining approximate solutions of the differential equations of motion for a wedge-shaped hull, making the additional assumptions that the hull is rigid, the water surface is smooth, the trim angle is constant during impact, and the wing lift equals the weight. Formulas are presented for maximum accelerations, maximum draft, and the draft at the instant of maximum acceleration. The theoretical results are then compared with experimental data from impact basin tests. The measured accelerations agree quite well with the theory; the measured drafts are consistently lower than the theoretical predictions.

A. H. Flax, USA

**1441. R. H. Miller, "Helicopter control and stability in hovering flight,"** *J. aero. Sci.*, Aug. 1948, vol. 15, pp. 453-472.

In the first part calculated stability and control response characteristics of several types of helicopters are compared with those of conventional aircraft. The generally poor dynamic stability characteristics of conventional unstabilized helicopters are shown to be due to lack of static stability and very low damping in pitch or roll.

Several means of improving the stability are considered: (a) Use of a servo actuated by a displacement gyro proportional to the angle of pitch or roll, providing static stability; (b) addition of a rate gyro in system (a) to provide damping; (c) elimination of blade flapping through use of a rigid rotor; (d) use of large

blade-flapping-hinge offsets to obtain a compromise between the advantages and disadvantages of flapping- and rigid-blade helicopters; (e) adjustment of blade center of gravity and aerodynamic center relative to the blade-feathering axis.

System (a) is shown to give poor initial control response and to be subject to instability again if servo time lags are not small enough; (b) seems promising, but also decreases controllability; (c) decreases the instability, but does not provide complete stability and is probably not attainable for large rotors, due to blade flexibility; (d) does not provide complete stability, but improves both stability and controllability; it may involve serious vibration difficulties; (e) offers a wide range of compromises between stability and controllability, but with numerous complications in the practical case, due to blade torsional flexibility and stick forces and vibrations which must be balanced between blades.

The second part deals with the derivation and methods of solution of the equations of motion, including four degrees of body freedom and two degrees of blade freedom in flapping (longitudinal and lateral flapping constants). Certain errors and omissions in earlier literature on the subject are corrected. In the third part the equations of motion are solved for the various cases mentioned. It is shown that in many cases a single plane analysis, involving only three degrees of freedom, with the rotor motion treated as quasi-static [Hohenemser, *Nat. adv. Comm. Aero. tech. Memo.*, no. 907] is a sufficiently good approximation. Servo controls, including the effects of a simple time constant, are analyzed by means of the frequency response method.

A. H. Flax, USA

## Aeroelasticity (Flutter, Divergence, etc.)

(See Rev. 1374)

## Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1375, 1411, 1441, 1450)

**1442. H. Tahsin Oenalp, "The elliptic law in the calculation of steam and gas turbines"** (in German), thesis, Eidgenössische Technische Hochschule, Zurich, 1948, 94 pp.

Stodola found experimentally that the curve representing the steam flow through a steam turbine at constant initial pressure and constant rpm but variable back pressure is an ellipse. G. Flügel showed that with simplifying assumptions this law could be established theoretically. Since then this law has been used for many important calculations in the design of steam and gas turbines, often with far-fetched assumptions.

The author discusses at great length the applicability of this law in the design of turbines. In the first chapter he makes exact calculations for the flow of ideal and real gases and vapors through a single orifice and compares the results with the assumed ellipse law. Finding good agreement, he investigates the applicability of the results obtained to successive stages of a turbine, taking into account the transfer of kinetic energy from one stage to the next. Finally, he discusses the difficult problem of flow through radially long turbine blades to test the applicability of the law in this case. Several practical problems are worked through in detail.

Karl E. Schoenherr, USA

**1443. F. B. Gustafson and G. C. Myers, Jr., "Stalling of helicopter blades,"** *Nat. adv. Comm. Aero. Rep.*, no. 840, 1946 (publ. in 1947), 6 pp.

An experimental flight investigation of helicopter-tip stall was carried out. To indicate stalling, one rotor blade was fitted with

tufts, to photo spread manner the he angle effects helicop

1444 high sp 1948, 2 This various suitabl figures Mach for pr

1445 tests, This wall ef forces of a r setting sidewa The sle tion f height

1446 electri 1948, 4 Whe pressil are rec top an culation and pr metho discuss experim

1447 ductio 77. The fers be second ity wh The au and sh better the pap dynam

tufts, and a motion-picture camera was placed on the rotor hub to photograph these tufts. It was found that stall occurred and spread over the rotor blade with increasing speed, roughly in the manner predicted by the usual theory for flapping rotors. For the helicopter used in the tests, it was observed that when the tip angle of attack was increased 4 deg above its initial stall angle, the effects of the stall were severe enough so that the pilot deemed the helicopter uncontrollable.

A. H. Flax, USA

## Flow and Flight Test Techniques

1444. G. M. Lilley, "Some notes on the performance of small high speed wind tunnels," *Coll. Aero. Cranfield Rep.*, no. 23, Dec. 1948, 25 pp.

This paper discusses briefly the layouts and performances of various types of tunnels using less than 100 hp which might be suitable for research and demonstration at colleges. Performance figures are based on a 2-in-square test section and a range of Mach number from 0.5 to 4. Some simple calculations required for preliminary design are discussed. J. S. Isenberg, USA

1445. J. Sanders, "Wind tunnel corrections in ground-effect tests," *Nat. Res. Coun. Can. Aero. Rep.* no. AR-5, 1948, 26 pp.

This paper considers the incompressible flow corrections for wall effects which should be applied to the measured aerodynamic forces in the case where the model is mounted close to the floor of a rectangular tunnel. Previous results for upwash and tail-setting corrections are reviewed. Axial-velocity corrections and sidewash corrections are deduced, using the method of images. The slow convergence of the infinite series involved in these correction formulas is improved. Numerical results for a tunnel of height-breadth ratio of 0.35 are given. J. S. Isenberg, USA

1446. J. Sanders, "Wall corrections for a two-dimensional electrical tank," *Nat. Res. Coun. Canada Engng. Rep.*, MA-213, 1948, 40 pp.

When the electromagnetic analogy is used to represent incompressible potential flow about a cylinder in free air, corrections are required for the presence of the front and rear as well as the top and bottom walls of the tank. Assuming an airfoil with circulation, the author calculates the corrections to lift, moment, and pressure coefficient on a flat plate, by the usual approximate methods used for wind-tunnel wall interference. He does not discuss the technique of establishing the circulation in actual experiments.

W. R. Sears, USA

## Thermodynamics

(See also Revs. 1417, 1429, 1442, 1454, 1456, 1459)

1447. R. C. Tolman and P. C. Fine, "On the irreversible production of entropy," *Rev. mod. Phys.*, Jan. 1948, vol. 20, pp. 51-77.

The study of many irreversible processes involving heat transfers becomes possible when the entropy change according to the second law of thermodynamics is written in the form of an equality which includes the entropy increase due to irreversibilities. The authors review the applications which this method has found, and show that it has the additional advantage of leading to a better understanding of all irreversible changes of state. Thus, the paper is a brief but comprehensive introduction to the thermodynamic analysis of irreversible processes.

Among the applications selected, some of which are the authors' original contributions, are: an equation for the efficiency of irreversible production of work, which serves to examine power cycles as well as steady-state flow and chemical processes; an analysis of transition through a shock wave; nonisothermal flow of a compressible, viscous fluid; and thermoelectricity. In a discussion of the validity of the temperature concept under non-uniform temperature conditions, the authors arrive at the conclusion that, because the empirical temperature concept becomes ambiguous, a specific definition of temperature adapted to the prevailing nonequilibrium conditions is necessary in order to justify the use of thermodynamic theories in such cases.

E. F. Lype, USA

1448. J. M. Gilchrist, "Chart for the investigation of thermodynamic cycles in internal combustion engines and turbines," *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 43, pp. 335-349.

The author presents a graphical method of analysis of the thermodynamic processes associated with internal-combustion engines. A chart with a linear scale of temperature as the ordinate and a linear scale of internal energy as the abscissa, is used together with a horizontal logarithmic scale for volume ratios and a vertical logarithmic scale for pressure ratios. The chart construction allows for variable specific heats, for dissociation, and for variable compositions.

The graphical method of analysis was applied to a general case which embraces the Otto, Diesel, Atkinson, and Humphrey processes. By means of this graphical analysis one can determine the work and efficiency of the device. This method is applied also to gas-turbine processes.

The author claims that this graphical method is very useful for instructional purposes; but there are many teachers who would take issue with him on this point after close inspection of his complicated diagrams.

Joseph Kaye, USA

1449. V. N. Huff and C. S. Calvert, "Charts for the computation of equilibrium composition of chemical reactions in the carbon-hydrogen-oxygen-nitrogen system at temperatures from 2000° to 5000° K," *Nat. adv. Comm. Aero. tech. Note*, no. 1653, July 1948, 43 pp.

These charts are designed to facilitate the solution of the system of higher-order algebraic equations encountered in the analysis of the equilibrium composition of the products of simultaneous chemical reactions. The charts apply to reactions occurring in the combustion of liquid and solid rocket propellants with pure oxygen, and in the explosion of gun powders where temperatures are high enough to promote dissociation of  $H_2O$ ,  $H_2$ , and  $N_2$ , and formation of  $NO$ . The authors develop a method of successive approximations, in which the system of simultaneous equations is written in such a manner that it becomes solvable when two of the unknown quantities are estimated, the correct solution being obtained after a few steps. The initial estimate and calculation of the remaining unknowns are done with the aid of the charts.

Eric F. Lype, USA

1450. Peter Lloyd, "The fuel problem in gas turbines," *Proc. Instn. mech. Engrs.*, 1948, vol. 159, pp. 220-229.

This is a survey of the properties of hydrocarbon fuels with regard to the combustion process in gas turbines. The author shows that a number of fuel properties including carbon-hydrogen ratio, density, vapor pressure and inflammability limits can be correlated with the combustion behavior of the fuel. A detailed discussion of these properties is presented, indicating that vis-

osity, volatility, analysis of the organic constituents and the characteristics of the ash are the most significant ones from the point of view of gas-turbine operation. With the aid of these characteristics as determined by laboratory tests it should be possible to predict the performance of new fuels.

Andrew A. Fejer, USA

**1451. Scott E. Wood, "The entropy of mixing of binary liquid mixtures,"** *J. chem. Phys.*, June 1947, vol. 15, pp. 358-363.

An analysis is made of three factors which may contribute to the entropy of mixing of binary solutions in excess of that for ideal solutions. These factors are: the spatial distribution of the molecules about a given reference molecule, the relative volumes of the molecules, and the lack of randomness of the orientational distribution of the molecules about a reference molecule. The spatial distribution has the least effect; it yields a negative value for the excess entropy and becomes zero at infinite dilution. The effect of differences in the volumes of the molecules of the components is positive and may be appreciable. The greatest effect is obtained from the orientational distribution; the difference is positive and is comparable, at least in order of magnitude, to the observed partial molal excess entropy of mixing at constant pressure and infinite dilution for the six binary mixtures composed of benzene, carbon tetrachloride, cyclohexane, and methanol.

The contribution of the orientational distribution is considered to be the result of two effects, one due to the orientational distribution of the solute becoming completely random when dissolved at infinite dilution, and the other due to the orientational distribution of the solvent being changed by the substitution of a solute molecule for a solvent molecule. The individual behavior of each solvent is evident and is consistent with its molecular structure.

J. Howard Childs, USA

**1452. Malcolm Dole, "Statistical thermodynamics of the sorption of vapors by solids,"** *J. chem. Phys.*, Jan. 1948, vol. 16, pp. 25-30.

The statistical method of treating the sorption of vapors, begun by Fowler and Guggenheim and extended to multilayer sorption by Cassie and Hill, is further generalized to include variable heats of sorption in different layers. By adsorption is meant adhesion of the vapor on a free surface where the gas molecule can collide directly with the adsorbing site. By sorption is meant the increase in weight of the solid as a result of the penetration of water vapor within the solid at sorption sites where there is no free surface.

A general partition function is set up in terms of the number of sorption sites, the relative molecular populations, and the number of molecular layers. Special solutions of the partition function are then obtained, such as Raoult's law, the Langmuir sorption isotherm, and the Brunauer-Emmet-Teller sorption isotherm. The treatment is also extended to the case of multilayer sorption with interaction between sorbed molecules in the same layer.

Joseph Kaye, USA

## Heat Transfer; Diffusion

(See also Rev. 1434)

**1453. Th. E. Schmidt, "Heat transfer calculations for extended surfaces,"** *Refrig. Engng.*, Apr. 1949, vol. 57, pp. 351-357.

To facilitate calculation of heat transmission from and to finned plane surfaces and finned tubing, the apparent coefficient of heat transfer is introduced as a function of the dimensions, the true mean coefficient of heat transfer, and  $\gamma = \theta_E/\theta$ , called the degree

of fin efficiency, where  $\theta_E$  and  $\theta$  are the mean temperature differences between the extended and the prime area respectively and the ambient. This concept of the degree of efficiency is very important, and simple formulas are given for calculating it for shapes most important in actual practice. Many questions can be clarified by calculations based on  $\gamma$ , even when the mean coefficients are not accurately known. Evaluation of test data has revealed that the coefficient of heat transfer is greatly dependent on the relation of height of fin to spacing. A formula for a simple relation is given.

The interrelations found have been employed for checking the economic value of fins of certain dimensions. Dimensional optima are calculated for capacity vs. unextended surfaces, vs. face area, vs. volume of space, vs. weight and vs. cost. The fact is revealed that certain designs satisfy several of these optima simultaneously. Results of calculations made for certain important cases are represented in diagrams.

Alexander Mendelson, USA

**1454. I. Amdur, "Low temperature transport properties of gases. I. Helium,"** *J. chem. Phys.*, July 1947, vol. 15, pp. 482-487.

Values have been computed for the viscosity, thermal conductivity, and coefficient of self-diffusion of gaseous helium for temperatures below 200 K by application of classical scattering theory within angular regions where diffraction effects are absent. Values of classical total-collision cross sections are obtained which are used to evaluate classical cross sections for viscosity, thermal conductivity, and self-diffusion appropriate to a Maxwellian gas. Numerical values of the transport properties are obtained by substitution of the appropriate classical cross sections into the exact transport-property formulas of Chapman and Enskog.

In the region 200-14 K, the average absolute deviations between calculated and experimental values are 1.9 per cent for viscosity and 4.6 per cent for thermal conductivity. Extrapolation of calculated viscosity values to 1.64 K appears to be justified on the basis of agreement with experimental values. It cannot be stated that similar extrapolation in the case of thermal conductivity is valid because of possible uncertainties in the experimental values between 3.95 and 1.62 K. Values of the self-diffusion coefficient have been calculated between 200 and 5 K, but experimental values are not available for comparison.

Stuart R. Brinkley, Jr., USA

**1455. R. Gregor, "Winter heat from summer and waste energy, and calculation of heat losses of underground water as a heat reservoir"** (in German), *Schweiz. Arch.*, Feb. 1949, vol. 15, pp. 36-48.

The heat losses and thermal efficiency of a heat reservoir using underground water for the storage of summer heat and waste energy is investigated. In the scheme described by the author, the underground water is heated during the summer months by pumping it to ground level, passing it through a heat exchanger and returning it to the reservoir. A fraction of this heat is made available in the winter months by adoption of a similar procedure.

The boundaries of the reservoir are formed by one of three methods: (1) natural impermeable soil layers, (2) artificial walls formed by injection of clay, and (3) a hydrodynamic limiting flow. The latter depends on the rate of flow of the circulating water, the number and positioning of the supply and outlet pipes connecting the reservoir to the heat exchanger, and the natural flow velocity and direction of the underground stream. Except in extremely favorable circumstances, a hydrodynamic limiting flow is unlikely to be satisfactory in practice.

The e  
for the f  
graphica  
fer is ob  
and (3)  
sible th  
model ex  
using sin  
ity of flo  
of the li  
The t

is show  
stallatio  
ciency is  
meters  
the oute  
be redu

**1456.**  
(in Fre  
930-942  
In de  
the mol  
of diffus  
Apply  
fluenced  
eral coe

**1457.**  
actance  
1949, v  
In or  
ment h  
varying  
thickne  
1000 ep  
more d  
effect w  
low am  
calcula  
been sh  
of the l  
comes  
By assu  
the act

**1458**  
crystals  
Meas  
salt ne  
using a  
Huntin  
The  
determ  
pressio

The equations of continuity, motion and energy are formulated for the flow of water through a permeable soil. An approximate graphical solution for the temperature distribution and heat transfer is obtained. The heat loss due to free convection in cases (2) and (3) is also investigated. It is stated that where possible the theoretical calculations should be supplemented by model experiments. A dynamically similar model of the reservoir using similar soil and water can be constructed in which the velocity of flow in the model is reduced in the same proportion as that of the linear geometric scale.

The thermodynamic efficiency defined as the ratio

$$\frac{\text{Quantity of heat in the reservoir}}{\text{Quantity of heat fed into the reservoir}}$$

is shown to be over 90 per cent in certain carefully designed installations using artificial walls. The maximum thermal efficiency is obtained for large and deep reservoirs (i.e.,  $1 \times 10^6$  cubic meters at 30 meters). For small reservoirs the permeability of the outer soil must be low, otherwise the thermal efficiency may be reduced to zero.

G. M. Lilley, England

**1456. I. Prigogine, "On the role of mass velocity in diffusion"** (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 12, pp. 930-942.

In defining the mean velocity of a mixture it is possible to use the molar, the weight and the volume average. The coefficient of diffusion must be defined differently in each case.

Applying the principle that the entropy increase is not influenced by the choice of variables, the relations between the several coefficients of diffusion are derived.

H. C. Brinkman, Holland

## Acoustics

(See also Revs. 1362, 1395, 1396)

**1457. R. H. Bolt, S. Labate, and U. Ingård, "The acoustic reactance of small circular orifices,"** *J. acoust. Soc. Amer.*, Mar. 1949, vol. 21, pp. 94-97.

In order to determine the effective mass of an orifice, a measurement has been made of the impedances of a number of orifices varying in diameter from 2 cm down to 0.357 cm, with diameter-thickness ratios from 4 to 40, over a frequency range from 200 to 1000 cps. The resistance component, which will be discussed in more detail in a subsequent paper, appears to show a nonlinear effect with amplitude as also does the reactance component. For low amplitudes the effective mass has been compared with that calculated from the assumption of plane piston motion and has been shown to depart from this considerably. By taking account of the higher-order modes that can be set up, the agreement becomes considerably better, particularly for wide-diameter tubes. By assuming that the effective radius is somewhat smaller than the actual radius, a still better agreement is obtained.

Warren P. Mason, USA

**1458. W. J. Price, "Ultrasonic measurements on Rochelle salt crystals,"** *Phys. Rev.*, Mar. 15, 1949, vol. 75, pp. 946-952.

Measurements of the anomalous elastic properties of Rochelle salt near the upper Curie temperature (about 24°C) were made, using a pulse technique at 10 mc per sec following the method of Huntington [*Phys. Rev.*, 1947, vol. 72, p. 321].

The temperature dependence of the following quantities was determined: the foiled compliance  $1/S_{44}^E$ , the velocity of a compressional mode in a 45-deg X-cut crystal, and the sound attenua-

tion at 10 mc per sec for the above mode and for a transverse mode along the Z-axis. The attenuation is due mostly to dielectric losses, and these were separated from other losses using auxiliary measurements along directions which avoid electromechanical coupling.

The velocity in a mode associated with  $S_{44}^E$  depends on the field strength in the X-direction; this effect is shown to vary also with the polarity of the bias field. Such unipolarity is not predicted by the interaction theory and is not revealed by self-resonance methods of measurement. Evidence is presented to show that the unipolarity is a "morphic" effect produced by the reduction in crystal symmetry produced when the lattice is sheared.

Martin Greenspan, USA

## Ballistics, Detonics (Explosions)

(See also Rev. 1449)

**1459. Melvin A. Cook, "Fugacity determinations of the products of detonation,"** *J. chem. Phys.*, Nov. 1948, vol. 16, pp. 1081-1086.

By employing the equation of state derived from the hydrodynamic theory and observed detonation velocities, the thermodynamic equilibrium constant  $K_p(T)$  is derived in terms of the concentrations of the various gases comprising the products of detonation and the covolume  $a(v)$ , which may be evaluated for any known or assumed density of the products of detonation. A reiteration method is developed for solving simultaneously as many equilibrium relations as is necessary to define completely the composition of the products of detonation. The application of the theory is illustrated by calculations for TNT, which are compared with experimental data as well as with results obtained from the approximate partial-pressure method. Calculations are presented for several additional explosives.

J. Howard Childs, USA

## Soil Mechanics, Seepage

(See also Revs. 1410, 1470)

**1460. Gregory P. Tschebotarioff, "Large scale earth pressure tests with model flexible bulkheads,"** Edwards Bros., Inc., Ann Arbor, 1949, 112 pp., 105 figs., \$1.

This report of tests made at Princeton University is a rich source of data. Displacements and SR-4 strain gage measurements were made on the middle one of three vertical sections of model bulkheads backfilled with various materials. Detailed test results are presented for three tests to illustrate the refinement of the data and the procedure used in deriving the bending moments and pressures.

The first series of tests used 1:5 models of an anchored 30-ft wall with hinged bottom. They showed that a sand dike at natural slope interposed between the bulkhead, and a fluid-clay backfill gave the same pressures as a backfill entirely of sand. The lateral pressure ratio of initially fluid-clay backfill reduced to 0.5 during consolidation with no outward movement of the bulkhead, but reduced no further when outward movement of the wall was induced. This was also demonstrated in a laboratory device.

A method of using the electrical strain gages under water was developed and used in a second series of tests at 1:10 model scale, with the bottom of the piles in sand to eliminate the restraint of the tank bottom evidenced in the first series. The results of 20 tests are presented in considerable detail. While the tests showed that the yield of the anchor eliminated vertical arching when the

bulkhead was backfilled with sand, the bending moments were decreased by transfer of active pressures to soil beneath the dredge line and the fact that the resultant of passive pressures was higher than previously believed. It was found that full wall friction could be developed for passive pressure. Severe vibration behind or in front of the wall changed the pressures but did not increase the bending moment. On the basis of the test results the following design procedure for bulkheads in sand is proposed: Depth of embedment equals 0.43 times the height and a hinge at dredge level is assumed; for anchor pull an earth-pressure ratio of .29 may be used, while for bending moment the earth-pressure ratio may be reduced somewhat for passive pressure above the anchor.

Since the test results refute some widespread hypotheses explaining certain previous observations, the compatibility of the present test results with these same observations is demonstrated. The author emphasizes that no tests were made with undisturbed clays or with relieving platforms, suggests that high priority in further research be given to full-scale observations, and gives some recommendations concerning procedures for field observations.

Edward S. Barber, USA

**1461. H. Q. Golden, "Measurement of pressure in timbering of a trench in clay"** (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 2, pp. 76-81.

This paper describes some measurements of the pressure in the timbering of a deep excavation behind a retaining wall in the London clay. A gage length of 5 ft on a strut was measured by means of a specially developed micrometer, reading to 0.01 mm. The results indicate that the pressure distribution approximates a parabola instead of a triangle.

Rollie G. Fehrman, USA

**1462. O. K. Peck and Ralph B. Peck, "Earth pressure against underground constructions. Experience with flexible culverts through railroad embankments,"** *Proc. Sec. int. Conf. Soil Mech. Found. Engng.* 1948, vol. 2, pp. 95-98.

Diameter changes in a number of flexible corrugated-steel plate culverts of 7.5 to 15 ft diam were measured over periods from 2 to 6 years, beneath railroad-embankment fill of selected soil varying in depth from 2 to 50 ft. The factors affecting the field performance of such culverts are outlined. Emphasis is placed on proper soil compaction. The vertical diameter of the culverts was found to shorten while the horizontal diameter increased. These deformations are believed sufficient to develop soil pressures practically equal in all directions around the entire structure, bending moments due to differences in the intensity of vertical and lateral pressure being ignored in the design.

G. P. Tschebotarioff, USA

**1463. J. Jáky, "Stability of earthworks in the plastic stage"** (in French), *Publ. tech. Univ. Budapest (Műegyetemi Közl.)*, 1948, no. 3, pp. 158-172.

This is the last of a series of studies on the title subject (see Revs. 852, May 1948, 801, June 1949).

In it the author deals with the various stress conditions prevailing in a heavy earth mass, by the help of polar coordinates. Extending the special solutions of Nádai, he proves that, besides slip lines in the form of concentric circles, other circular slip lines are possible with centers located on an arbitrary geometric line, and with radii which are a function of the coordinates of the centers. From the conditions of equilibrium he succeeds in writing down the partial differential equation and its general solution (Equations 135, 136).

This solution is illustrated by two practical examples. One presents the slip lines around a hole in a plastic body and the corresponding stresses acting on the surface of the hole. The other shows the stress conditions of an earth mass loaded by uniformly distributed oblique loads.

Ch. Széchy, Hungary

**1464. F. L. D. Wooltorton, "Relation between the plastic index and the percentage of fines in granular soil stabilization,"** *Proc. Highw. Res. Bd.*, 27 Ann. Meet., 1948, pp. 479-490.

Specifications for granular soil stabilization usually consist of maximum permissible values for the plastic index (*PI*) of the fraction *f* passing the No. 40 sieve. A discussion of the physical background of these empirical specifications leads the author to the conclusion that for stability of a granular material the amount of moisture absorbed over the plastic range should not exceed the pore space available.

A discussion of engineering experience is summarized in the rule  $(PI) \times f = C$ , where *C* is a constant, which however in some cases appears to depend on the percentage of soil fines.

H. C. Brinkman, Holland

## Geophysics, Meteorology, Oceanography

(See also Revs. 1350, 1378, 1412)

**1465. Robert E. Horton, "The physics of thunderstorms,"** *Trans. Amer. geophys. Un.*, Dec. 1948, vol. 29, pp. 810-844.

A special type of thunderstorm is discussed, called by the author "tubular," in which the entrainment of air from the environment supposedly acts only in a shallow layer near the ground. The air then ascends through a tube without any mixing with the surrounding air and spreads around this core in an outflow layer. From the equation of continuity under highly simplified assumptions the dimensions of the convective core and the thicknesses of the different layers can be determined, as well as the ascending velocity in the core.

The average distribution and intensity of rain in a thunderstorm is thoroughly discussed in relation to this model. The rain falling through the core shows different characteristics from the rain falling from the peripheral belt in the outflow layer. By assuming these two different types of rain, the author makes it possible to reasonably interpret some observed phenomena, like lightning and wind gusts in a thunderstorm.

Z. Sekera, USA

**1466. Robert E. Horton, "Convictional vortex rings—hail,"** *Trans. Amer. geophys. Un.*, Feb. 1949, vol. 30, pp. 29-45.

Besides the well-known thunderstorm system with the main convective flow through a vertical pipe, thunderstorms and hailstorms are sometimes observed with a different hydrodynamic system, in which a series of vortex rings is projected vertically upward. The ring sections themselves move with a constant angular velocity. The ascending moist air is concentrated in the ring sections forming a big cumulus cloud. Whether tubular convection or a vortex ring system develops, depends on the conditions. In cases of a violent, abrupt and discontinuous uprush of the air through a stable layer, as many vortex rings will form as there are impulses.

A certain experimental verification can be given by using a smoke box and producing a series of vortex rings. The rings travel upward as if their sections were being rolled forward by the core. There is relatively little frictional resistance, i.e., the vortices can travel a long distance on their initial energy, before they

are broken up by viscous resistance. Following the smoke box analogy a theory of the phenomena is given. The conditions in such cumuli are very favorable for the formation of hailstones of uniform size and structure, which are sometimes observed. A theory on hailstorms is, therefore, added. The paper has been edited by R. Van Vliet as a posthumous publication.

Horst Merbt, Germany

**1467. Bertram Darell Richards, "Tidal power: Its development and utilization,"** *J. Instn. civ. Engrs.*, Apr. 1948, vol. 30, pp. 104-109.

This paper gives a comprehensive survey of the problems of tidal power, mainly from the civil-engineering aspect. The survey is necessarily limited by the fact that tidal power, apart from very small schemes, exists on paper only, economic considerations having prevented the execution of any large scheme up to the present date. The increase of price of coal and the fact that tidal power operating costs are independent of market fluctuations increase considerably the importance of these investigations for the future. Bearing in mind these inherent limitations of subject, the author has succeeded in making a thorough and useful contribution to the art of power generation.

There is a brief introduction giving the historical background, followed by the basic facts of tidal phenomena. The author then describes the single-basin operation showing how the power obtained is intermittent and subject to large fluctuations. The problem of regulating this power to suit the consumer is considered in detail, with description of pumped storage and auxiliary power schemes. Double-basin schemes, giving a more nearly constant power, are illustrated by diagrams of time vs. power output and of general layout.

A description then follows of the four major tidal power projects that have recently received attention. The Severn Barrage single-basin scheme has been under consideration for a century and the author reviews the principal features of the 1944 Panel report mentioning the tidal model constructed by Professor A. H. Gibson. The Petitecodiac-Memramcook double-basin system takes advantage of two estuaries having an exceptional tidal range. On the Passamaquoddy project a summary follows of the single-basin and double-basin proposals. The two French schemes comprise the Rance project where the intermittent power could readily be accepted by the electrical system, and Mont St. Michel where the power proposed is so large as to demand some means of regulation, thus making the latter unsuitable for immediate development. A table of tidal range, power and costs for these four schemes is given.

The author concludes with a résumé of the pros and cons of tidal power and a reference to the future development of atomic power. There are twelve diagrams and eight references to published papers.

H. H. Anderson, Scotland

**1468. Tu-cheng Yeh, "On energy dispersion in the atmosphere,"** *J. Met.*, Feb. 1949, vol. 6, pp. 1-16.

The problem of energy propagation by dispersive one-dimensional waves is discussed for four different atmospheric models distinguished according to the characteristics of their upper boundary (rigid boundary for nondivergent flow, free surface for divergent flow) or of the horizontal distribution of density (with or without a constant north-south density gradient). The relationship between the group and the phase velocity is first studied for these models, and the group velocity is in all cases larger than the phase velocity, making the formation of new waves possible ahead of initial waves. The presence of convergence or divergence and of horizontal solenoids (density gradients) makes possible the

formation of waves with negative group velocity, that is, of upstream energy propagation. But only in the case of existing solenoids is the negative group velocity larger than the phase velocity and is the formation of new waves upstream possible.

In the next part the phase distribution is discussed in a wave motion due to a point-source disturbance. In a diagram with  $t$  as ordinate and  $x$  as abscissa, the constant-phase curves can be drawn and give all information about wave motion (except the amplitude), in particular the number of different wave trains and the direction of wave motion for particular wave lengths. The various features of such a motion in different atmospheric models are easily understood from a set of such diagrams.

A more general type of energy distribution is then studied for two special cases: (a) the injection of positive vorticity (cyclogenesis) along a particular longitude into a straight basic current of constant speed, and (b) the dispersion of an initial solitary wave connected with a pressure rise in a limited region. In the first case a ridge develops immediately east of the source of cyclogenesis, followed by an establishment of a ridge further to the east with a trough between these two ridges, in full agreement with synoptic evidence. After a sufficiently long time a steady-state wave motion develops, identical for all the models considered.

In the second case the process of energy dispersion of a solitary wave is quite different for different latitudes. In low latitudes the dispersion progresses very rapidly with a slight shift of the high pressure center westwards. In higher latitudes the dispersion progresses much more slowly, and the shift of the center westwards is quite apparent with an appreciable decrease of initial pressure (1/7 during 48 hours at 40 deg lat). However in the vicinity of the pole the westward shift is quite slow, and the decrease of the pressure in the center is much less. The westward shift of the initial anticyclone corresponds to the observed blocking effect, and the discussion of energy dispersion thus offers the first theoretical explanation of this synoptically important phenomenon. From the discussion it follows that the blocking effect is largest at high latitudes, while the speed of the blocking wave decreases and the intensity of the wave and its lifetime increases with the latitude.

Z. Sekera, USA

**1469. Athelstan F. Spilhaus and Arthur R. Miller, "The sea sampler,"** *J. Mar. Res.*, Nov. 1948, vol. 7, no. 3, pp. 370-384.

An improved model of the thermographic sea sampler is described. It consists of twelve bottles attached to a conventional bathythermograph and arranged to trip at various depths which can be chosen at will. This device enables samples to be taken along with temperature soundings while the ship is under way. Observations made with the sea sampler were compared with those made with Nansen bottles, and it was concluded that errors in the former were due to throttling lag of the sampler bottles. The theory of this lag is developed and a method of correcting for it is presented. The device appears to be a very important addition to the instruments available for oceanographic observations.

M. Neiburger, USA

**1470. E. Bucher, "Contribution to the theory of avalanche protection"** (in German with French and English summaries), thesis, Eidgenössische Technische Hochschule, Zurich, 1948, 114 pp.

Different means of preventing avalanches are systematically surveyed and their various features described; a rational procedure is given by which appropriate structures may be designed. This survey is preceded by a thorough study of snow as a material, and of its behavior in typical situations. Since this part of the work occupies two-thirds of the space, "the physical properties of

snow and of snow-banks" might well be an auxiliary title. And at the same time the meteorologic and hydrologic features of snow fall, accumulation, packing, wastage, and other progressive changes in the snow cover are also studied.

In a remarkable series of charts relating to conditions at the standard test field at Weissfluhjoch, daily records of meteorological data are plotted, and at intervals of about 3 weeks the properties of the snow itself on increments of 10 cm in depth. Such a chart is shown for each of the years 1936 to 1946.

A great number of different physical properties of snow are considered and the methods and results of laboratory tests described. Special consideration is given to the laminated structure of the snow cover, and variability from layer to layer may be very great. For example, the viscosity of dry snow varies from  $10^6$  to  $10^{11}$  kg sec per  $m^2$ .

Among the types of structure, emphasis is given to *Rechen* (obstacles to snow movement set normal to the slope). In the last 8 pages, a method for estimating the load on such a structure is given in detail.

The 236 references to technical literature include a few items in general rheology, soil mechanics, physics of solids, and the like, but most of them refer to the specific topic, on which the coverage is thorough.

W. P. Roop, USA

## Lubrication; Bearings; Wear

(See also Rev. 1392)

1471. Hanns Herbert Ott, "Cylindrical journal bearings under nonstationary loading" (in German), thesis, Eidgenössische Technische Hochschule, Zurich, 1948, 103 pp.

In the first part of this paper a general differential equation for a journal bearing under a load which may vary in magnitude and direction is derived. The usual assumptions are made of perfect geometrical surfaces, negligible inertia forces, no axial flow, thinness and hence negligible curvature of the oil film, constant viscosity, and incompressible laminar flow following Newton's law of viscous flow. Since derivation of a general expression for the resultant force on the journal proves too difficult mathematically, several special cases are discussed.

The procedure adopted in discussing these is the inverse of the problem usually confronting the designer. The journal center is assumed to move in a given path and the resultant dynamic load is computed. This offers less mathematical difficulty than the more direct approach of specifying the loading and computing the resulting motion of the journal center. The special cases considered include a journal executing a radial sinusoidal motion and a journal traveling in an elliptical path with double harmonic motion. Several representative polar load diagrams are computed for these cases when the journal eccentricity is relatively small.

In a second part the motion of the journal center is computed for sinusoidal forces of small amplitude and constant direction.

In checking the derivation of the initial differential equation this reviewer has encountered certain difficulties. The author seems to have incorrectly applied the volume continuity equation of fluid mechanics, using this law in its integrated rather than in its differential form. If this is corrected, Equation [8] would read

$$\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) - 6 \eta (V_1 - V_2) \frac{\partial h}{\partial x} - 6 \eta h \frac{\partial (V_1 + V_2)}{\partial x} - 12 \frac{\partial h}{\partial t} = 0,$$

which differs from the author's equation in the sign of  $V_2$  in the second term, and in the presence of the term involving  $\partial(V_1 + V_2)/\partial x$ . The author apparently discarded the latter term, reasoning that neither  $V_1$  nor  $V_2$  will change along the bearing surface. However, in the dynamically loaded bearings subsequently discussed, the journal center translates in a curved path about the bearing center, the resultant velocities  $V_1$  and  $V_2$  then being functions of  $x$  so that the above term is generally not zero. It is unfortunate that these relatively trivial points should have to cast doubt upon the results of the outstanding mathematical analysis which follows.

Milton C. Shaw, USA

1472. G. Lundberg and A. Palmgren, "Dynamic capacity of rolling bearings" (in English), *Acta Polyt.*, 1947, no. 7, 50 pp.

This article gives the foundation for a unified theory of the dynamic capacity of rolling bearings. It is based on the hypothesis that the shear stress calculated by the Hertz theory may, through fatigue, produce a microscopic crack if it occurs in the vicinity of a "weak point" in the material. The distribution of these weak points throughout the material is statistical and may be taken as a material characteristic. Such a microscopic crack grows as a result of successive stress cycles, and if it reaches the surface, failure results. These assumptions are summarized analytically by the statement that the dispersion in life of the bearings is proportional to the stressed volume and is also some function of the maximum stress amplitude calculated by the Hertz theory, of the number of stress cycles and of the depth at which the maximum stress amplitude occurs.

This functional dependence is not analyzed in detail but is approximated by a power function of the three quantities involved. The three exponents are determined empirically from the dispersion in bearing lives observed in life tests, from the dependence of life on load, and from the dependence of the basic dynamic capacity on the bearing size for both point contact and line contact.

The dependence of the basic dynamic capacity (defined as the load which 90 per cent of a given set of similar bearings can endure for one million revolutions under either pure axial or pure radial thrust) of each element in a rolling bearing is then determined by considering the distribution of load among the rolling elements and extending the Hertz theory to take account of the relative conformity between these elements. The basic dynamic capacity of the various types of rolling bearings is then calculated from the capacities of each element for the applied radial and axial load components. Tables are given to facilitate such calculations for various types of bearings and loading.

John T. Burwell, Jr., USA

## Marine Engineering Problems

(See Revs. 1365, 1440)